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SITUATIONS, LANGUAGE AND LOGIC

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PREFACE

This monograph grew out of research at Xerox PARC and the Center for the Study of Language and Information (CSLI) during the first year of CSLI's existence. The Center was created as a meeting place for people from many different research traditions and there was much interest in seeing how the various approaches could be joined in a common effort to understand the complexity of language and information. CSLI was thus an ideal environment for our group and our enterprise.

Our original goal was to see how a well-developed linguistic theory, such as lexical-functional grammar, could be joined with the ideas emerging from research in situation semantics in a manner which would measure up to the technical standards set by Montague grammar. The outcome was our notion of situation schemata and the extension of constraint-based grammar formalisms to deal with semantic as well as syntactic information.

As our work progressed we widened our approach. We decided to also include a detailed study of the logic of situation theory, and to investigate how this logical theory is related to the relational theory of meaning developed in situation semantics.

We are pleased that the first version of this work (Fenstad et al. 1985) has been used as a basis for further studies at a number of different sites. We include here a sample of such work in Appendix A where Erik Colban of the Mathematics Department, University of Oslo, presents his analysis of locative prepositional phrases. We believe that his analysis is an illustration of the openness and extendability of our basic approach.

This book is a research monograph reporting on work in progress. We have restricted ourselves to basic facts of language and logic, and our aim has been to give a precise technical account of how to join linguistic form and semantic structure. We hope that the techniques for linguistic description which we illustrate here can be of wide applicability.

We are indebted to CSLI and the Xerox Palo Alto Research Center for the support of our research. We are grateful for the assistance in the preparation of the manuscript which we received from Dikran Karagueuzian and Lynn Ruggles at CSLI, and Denise Pawson at Xerox PARC. We also wish to thank Mr. Martin Scrivener of D. Reidel Publishing Company for his valuable advice.

CHAPTER I

INTRODUCTION

In this monograph our aim is to give an overall framework for relating the *linguistic form* of utterances and their *semantic interpretation* which is based on the idea of *constraint propagation.* In Chapter II we present an algorithm which converts linguistic form into a format, which we call a *situation schema*, better suited for semantic interpretation. In Chapter III we spell out the *meaning relation*, i.e. the interpretation of the situation schemata into a system of situation semantics. In Chapter IV we investigate the *structure of the semantic interpretation* through a hierarchy of many-sorted formal languages.

1 FROM LINGUISTIC FORM TO SITUATION SCHEMATA

The informational content of an utterance is determined not only by its linguistic form (i.e. its phonology, morphology and syntax), but also by a number of contextual factors. The interpretation of the utterance must satisfy all the constraints imposed by all the relevant aspects of the larger "utterance situation." Each semantically relevant aspect provides us with important, though partial, information about the interpretation of the utterance. There seems to be no firm empirical evidence that one component of linguistic form has primacy over the others in determining the interpretation, nor that the interpretation of the utterance proceeds in any particular order (e.g. the one suggested by the syntactic surface structure).

With this in mind we have chosen to represent the constraints which are imposed on the interpretation of the utterance by contextual and linguistic constituents of the utterance situation through a cumulative (or monotonic) system of constraint equations (Kay 1979; Kaplan and Bresnan 1982; Halvorsen 1983). Each equation makes a statement about properties of the interpretation, and the problem of systematically determining the interpretation becomes the problem of finding a solution to the constraint equations (Robinson 1965). The constraints flowing from various aspects of the utterance situation, whether contextual or formal, are all taken to be represented in the same equational form, which simplifies the integration of constraints from various levels.

In part one we are primarily concerned with the constraints derived from the linguistic form; in part two we discuss how larger aspects of the utterance situation constrain the meaning relation.

Our proposal contrasts with the traditional point of view that the meaning of larger phrases is determined through functional composition of word meanings where the control structure for the function composition is given exclusively by the syntactic structure of the phrase (Montague 1970). It is not our purpose to argue at length the deficiencies of the traditional point of view. But it does not seem to be the best interpretation strategy to "force" the full informational content of the utterance and the context through the syntactic tree to arrive at the meaning content.[1] Even restricting ourselves to linguistic form, syntax is not the obvious repository for information regarding the semantic impact of intonation or for resolving questions of scope or anaphoric reference. These points become even more pronounced for languages with free word order. In such languages, the syntactically motivated surface constituent structure is flat and lacking in the hierarchical organization required for the "logical syntax."

Taking certain liberties with traditional boundaries and hierarchies between linguistic modules does not imply disrespect. Obviously, there are natural syntactic and morphological (as well as other) clusters of constraint equations, and all the usual insights about the internal structure and interconnections of these components remain. Moreover, liberties should not degenerate into license. If a system of cumulative constraint equations is to acquire descriptive and theoretical "bite", we shall even-

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[1]Of course, by "packaging" enough contextual and interpretative factors into denotations, the letter of the compositionality principle can usually be obeyed after all, but often—in the process—its spirit has departed. Gawron (1986) suggests that the "descriptive content" of an utterance is determined exclusively by the syntactic analysis of the utterance, while its meaning depends on other factors as well. Our interpretations are largely coextensive with Gawron's "descriptive content," but we prefer a less syntax oriented approach (see Chapter III for supporting arguments).

tually have to study restrictions on the constraints that involve multiple components (e.g., syntax and semantics, or prosody and semantics). In our analysis the various components of linguistic description maintain their independence and the statement of generalizations on the semantic level does not affect the integrity of other levels.

We have chosen the format that we call *situation schemata* as a theoretical notion convenient for summing up information from linguistic form which is relevant for the semantic interpretation. In part one we present a grammar for a fragment of English; where situation schemata are derived through an extension to the traditional format of lexical functional grammar (see Kaplan and Bresnan 1982), i.e., context-free phrase-structure rules supplemented by constraint equations introduced both by the syntactic rules and in the lexicon. As a first step toward the semantic interpretation we seek consistent solutions to the set of semantic constraint equations. A solution, if it exists, can be presented in tabular form. A situation schema, in analogy with the notion of functional structure of the LFG-theory, is such a tabular representation. [2]

The notion of a situation schema also bears some similarity to the intuitive notion of "logical form." A simple declarative sentence may be taken to have a main "semantic predicate" relating a number of "actors" playing various "roles" (agent, patient, theme). Both the predicate and the roles actors play can be modified (by adjectives, adverbs, prepositional phrases, relative clauses). Accordingly, we shall take a situation schema to have the following basic format. Let φ be a simple declarative sentence, then:

[2]Although this monograph is indebted to the format and methodology of LFG, the present proposals are not dependent on the acceptance of specific linguistic theory—as will become clear in the following parts.

(1) Situation Schema of φ

$$\begin{bmatrix} SIT & — \\ REL & — \\ ARG.1 & — \\ \cdot & \cdot \\ \cdot & \cdot \\ ARG.n & — \\ LOC & — \\ POL & — \end{bmatrix}$$

This expresses that the situation schema of φ is a function with arguments *REL, ARG.1,..., ARG.n, LOC* (and possibly others; see section II. 4). The value of the function on the arguments *REL, ARG.1... ARG.n* is given by the "logical predicate" and the roles of the sentence φ. These values can either be simple, e.g. the value of an *ARG.i* can be the name of an actor or it can be the situation schema for a complex NP. In the latter case we get a new function or subordinate "box" as value determined by the structure of the NP (the determiner, the noun, the optional relative clause). Finally, the value of *LOC* is derived from the tense marker of the sentence; see section II.4 for a more precise description.

Evidently, "logical form" is a term with widely divergent uses in logic and linguistics; so much so that it has become meaningless, and in some quarters almost a pejorative. Nevertheless, we feel that the above level, intermediate between linguistic form and semantic reality, is worth displaying, as it exhibits the information conveyed by a sentence in a logically more perspicuous way while also providing a vehicle for describing anaphora and other phenomena for which some "representational level" is often proposed. To be more specific, the schemata depicted above themselves often leave interpretation under-determined. But they can be enriched by further instructions concerning anaphoric identities, or scope ordering of operators, which may eventually enforce unique readings. In this view, there lies also some similarity with the more idiosyncratic notion of "logical form" as used in transformational grammar. Note, however, that possession of "meaning" is not an all-or-nothing matter:

the original schemata certainly convey a lot of information, the additional mechanisms add more.

It is also possible to view situation schemata as a notation for situation-theoretic objects (i.e., states of affairs, parameterized states of affairs, types, etc.) with the attribute names of the situation schemata labeling the various elements of these structured objects. On this view the approach to semantic composition and the syntax-semantic interface presented here and in Fenstad et al. (1984) provide a detailed alternative to Gawron's (1986) proposals. See Chapter III.2 for details.

In Chapter II we arrive at situation schemata from both perspectives outlined above. First, we give a grammar which associates with each sentence of a fragment of English a set of equations, and we present Kaplan's algorithm (Kaplan and Bresnan 1982) for finding the solutions of such systems of equations. We also give a set of construction rules for situation schemata and we show how the two fit together: Each well-formed situation schema is the tabular representation of a consistent set of equations, and, conversely, each solution set can be represented as a situation schema. Let us once more emphasize that our algorithm is directed from linguistic form to situation schemata, hence the primacy of the former over the latter.[3] The latter is for us a theoretical notation, a convenient way of summing up information flowing from the linguistic form and partially determining the semantic interpretation. (Whether this level has an independent psychological significance, as various authors have suggested for logical form, is at present not more than a pleasing speculation.) Two points follow:

- The notion of situation schema is open; it depends upon the underlying theory of grammar and is susceptible both to emendation and extensions; this allows us to exploit available linguistic insights freely.
- Situation schemata can be adapted to various kinds of semantic interpretations; we could interpret in some system of higher order intensional logic as in the Montague tradition (Halvorsen 1983), or we could give some kind of operational interpretation in a suitable programming language. This allows us to exploit logical insights freely.

[3]Actually, inverting our algorithm is an interesting mathematical issue, with potential applications—but this topic will not be pursued here.

As will have become clear by now, we view our proposals as being well within a long semantical tradition. For instance, the basic form of situation schemata resembles that of classical logic on the one hand (cf. Sommers 1982), and that of classical linguistics on the other (cf. Tesnière 1959). Moreover, in their additional fine-structure, these schemata have been inspired both by the "functional structures" of Kaplan and Bresnan (1982) and the "discourse representation structures" of Kamp (1984). Thus, although one might feel worried about the increase of "strategic discussions" in semantics (with a constant fund of empirical input), there is also consolation in the discernible convergence of these arguments.

A final source of inspiration to be mentioned concerns the interpretation, rather than the format of our situation schemata. In Chapter III, we have opted for an interpretation in a system of situation semantics. Some familiarity with Barwise and Perry (1983) will be assumed on the part of the reader, but our presentation will be reasonably self-contained.

Remark. Since the first version of this work appeared situation theory has undergone several changes both in substance, in notation, and in terminology. For an initial review and critique with an extensive reply from Barwise and Perry, see *Linguistics and Philosophy* Vol. 8, No. 1. Barwise has written a series of papers reworking the the foundations of the theory *The Situation in Logic* I-III (Barwise 1986; to appear a; to appear b). We have adopted a more conventional model-theoretic starting point in this work, but our approach is fully compatible with the current trend in situation theory. Our terminology also differs in places from the terminology used in some other works on situation semantics, but these are "surface" differences which do not touch on matters of substance.

2 INTERPRETING SITUATION SCHEMATA

Adopting an approach of situation semantics (Barwise and Perry 1983), to us, means two things: employing a certain "semantic format" for explaining meaning, viz. a relational one; as well as opting for a certain kind of model in the background, in our case, suitably partial "structures of situations". We start by explaining the first issue.

The meaning of a (simple declarative) sentence φ is a relation between an *utterance situation*, u and a *described situation*, s; we shall use the situation schema or schemata associated with φ, written symbolically as $SIT.\varphi$ to mediate the connection between u and s and write the basic meaning relation in the following form.

(2) $u[\![SIT.\varphi]\!]s$

Notice that in interpreting an utterance of φ in a context u there is a flow of information, partly from the linguistic form, which we have encoded or summarized in the $SIT.\varphi$ schema, and partly from other contextual factors, which we "read off" from the utterance situation u. These combine to form a set of constraints on the described situation s. s is not uniquely determined; given u and an utterance of φ in u there will be several situations s that satisfy the constraints imposed.

There may also be an "inverse flow" of information: from the fact that s is a described situation relative to $SIT.\varphi$ and some discourse situation u, we may obtain quite specific information about u (Barwise and Perry 1983).

In Chapter III, we spell out (i.e. give an inductive construction of) how a given u and a given $SIT.\varphi$ constrain or (partially) determine a described situation s. We shall preface the general construction by a series of examples which exhibits the kind of constraints imposed by linguistic form. But these examples will also show that linguistic form alone underdetermines the interpretation. We shall indicate how the larger utterance situation u may add extra information which decides e.g. questions of scope and anaphoric reference. In connection with definite descriptions we shall also discuss a loading-mechanism which allows us to draw the correct distinction between the attributive versus the referential use of definite descriptions. We are not so much pleading for a specific linguistic theory of, say, anaphoric reference; our aim has rather been to work out an overall framework in which various proposals can be stated and compared.

This relational semantic format of Barwise and Perry is actually a natural outcome of a long series of interpretative proposals. Tarski's original truth definition has a relational form, be it one "biased" towards situations described, because of the limitations of predicate logic. Gradually, (philosophical) logicians have come to study even more types of expression, including those which essentially involve aspects of the utterance situation, such as indexicals, tense, etc.. Thus, several current notations in tense logic stress the interplay between a "moment of speech" and a "moment of evaluation"—these being the reducts of u and s to that particular case.

Indeed, it seems reasonable to say that semanticists have agreed for a long time on the various ingredients that enter into meaningful uses of language: linguistic structure, relatively stable semantic background conventions, more speaker-dependent connections, etc.. Where they have differed, in notation, and sometimes also in substance, is in ways of "cutting up" the picture. For instance, the semantic format which has become current in Montague Grammar lets linguistic expressions denote suitable semantic entities relative to an "index package," consisting of a possible world, a point in time, plus additional items as needed. Even though, in the final analysis, such a schema is essentially intertranslatable with the one presented here (cf. section IV.1 for some details), we feel that the relational format has unmistakable conceptual and heuristic advantages.

The type of model to be adopted will be explained in part two and again in three. For the moment, a brief indication must suffice. Basically, we try to employ a kind of model structure which embodies some basic insights from situation semantics, while staying relatively close to traditional structures in logical semantics. There is conservatism here, as we want to stay attuned to classical insights and techniques. But there is also a genuine conviction that our simple perspective may eventually avoid the conceptual and mathematical problems surrounding earlier versions of the theory.

Situation semantics is grounded in a set of primitives:

(3) S:*situations*
Λ:*locations*
$\mathcal{D}$:*individuals*
R:*relations*

For our immediate purposes we can leave the ontological status of the primitives open. They could be cut out of reality by some act of individuation or they could be abstract entities introduced to study the truth conditions and "valid" inferential patterns in natural language reasoning. Be that as it may, in either case the set of primitives comes with some structure. This will be elaborated upon in Chapter IV. Some minimal requirement is that each relation in R comes with an arity, i.e. a specification of the number of argument slots or roles of that relation. For our introductory purposes we impose no structure on the sets of situations and of individuals. The set Λ of locations is or represents connected regions of space-time. Thus Λ may be endowed with a rich geometric structure, and, perhaps, should be, if we were to give an analysis of perception verbs, such as "seeing that," which correctly classify verb phrases describing spatio-temporal processes. For our limited purposes in this monograph we shall be much more modest and assume that the set Λ comes equipped with two structural relations:

(4) $\prec$ – *temporally precedes*
$\circ$ – *temporally overlaps*

In Chapter IV we discuss the structural properties of these relations.

Primitives combine to form facts which can be either located or unlocated. Let r be an n-ary relation, l a location and $a_1, \ldots, a_n$ individuals. Basic located facts will have the form

(5) $at\ l : r, a_1, \ldots, a_n; 1$
$at\ l : r, a_1, \ldots, a_n; 0$

The intended meaning of the first is that at location l the relation r holds of the individuals $a_1, \ldots, a_n$; the intended meaning of the second is that at location l the relation r does not hold of $a_1, \ldots, a_n$.

The basic format of unlocated facts is:

(6) $r, a_1, \ldots, a_n; 1$
$r, a_1, \ldots, a_n; 0$

with a reading analogous to the one above. In addition to these facts we have unlocated "atomic" assertions concerning the location structure:

(7) $l \prec l'$
$l \circ l'$

expressing the above-mentioned relations of temporal precedence and overlap.

A situation s determines a set of facts, but it is not in the set-theoretic sense a set of facts. This "intensional" character of situations is not so important for our present task of interpreting an extensional language fragment, but will be of crucial importance in the semantic analysis of attitudes.

The basic format connecting the primitives is in the located case:

(8) $in\ s :\ at\ l : r, a_1, \ldots, a_n; 1$
$in\ s :\ at\ l : r, a_1, \ldots, a_n; 0$

We read the first as: in s at location l the relation r holds of $a_1, \ldots, a_n$. There is an analogous reading of the second clause, as well as of the corresponding unlocated versions.

3 THE LOGICAL POINT OF VIEW

Situation schemata were compared to "logical forms" in our earlier discussion. Now, as is well-known, the logical forms employed in standard logic, such as formulas of first-order predicate logic or more complex formal languages, serve a dual purpose. On the one hand, they are a vehicle for interpretation, leading to logical studies of expressiveness, and eventually the whole subject of interplay between languages and models, i.e. *model theory*. On the other hand, their particular form has also proved useful in providing structural accounts of valid inference, i.e., in *proof theory*.

From a logical point of view, then, a systematic study of situation schemata under these two aspects would be quite interesting. In fact, similar suggestions have occurred with a certain regularity in the literature—with the alluring prospect of a "natural logic" closer to linguistic form. In this monograph, however, we have followed a more traditional, but still informative road. In part three, a hierarchy of formal languages is introduced, starting with a purely propositional one, continuing with a predicate-logical language, subsequently enriched with tense operators, ending up with rich many-sorted languages allowing direct reference to locations and, eventually, situations themselves.

These restricted formalisms will function as a kind of "semantic laboratory" for various semantic notions proposed in the situation semantics literature. Notably, we shall investigate the phenomenon of "persistence" of information, and provide a precise characterization of those cases where it occurs. Moreover, as usual, a formal systematic perspective suggests variations upon earlier informal themes. For instance, we shall study various types of semantic connection between statements, such as "strong consequence," "involvement", and several types of synonymy. Finally, as for the more proof-theoretic aspect, a complete axiomatization will be given for our two-sorted predicate logic. That is, we exhibit the complete inferential behavior of its traditional logical constants: connectives ('not', 'and', 'or') and quantifiers ('all', 'some'), with some new twists due to the present partial perspective. In addition, following the long-standing tradition in philosophical logic of describing inferential behavior of other types of expression as well, we take a look at some formal operators for tenses and temporal auxiliaries.

The above hierarchy is a rather traditional one, not necessarily in harmony with the structure of natural language itself—where, for instance, quantifier mechanisms seem more basic than propositional connectives. But, our oblique perspective, when used with care, will prove illuminating all the same; if only, because it enables us to take advantage of a fund of insights already developed in contemporary logic. And in section IV.4 we explain how this "traditional" approach is related to the analysis of meaning developed in Chapter III of this work.

Even without the above logico-semantical issues in the foreground, there is a good deal of interest to the situation structures as such, i.e., our "ontology". We introduced in section I.2 the four primitive domains of situation semantics, $S, \Lambda, \mathcal{D}, R$. We shall use M to denote the situation structure of these domains,

(9) $$M = < S, \Lambda, \mathcal{D}, R >$$

A structure is more than the basic domains. Each domain may carry their own internal structure: The set Λ comes with the relations $\prec$ and $\circ$ with basic properties which reflect that Λ is or represents connected regions of space-time. The other sets may also carry internal structure, e.g. one individual may be part of another, one situation may extend another, relations may combine to form relational products etc.. For the most part we shall assume minimal internal structure on the various sets of primitives; in Chapter IV we discuss various options in more detail.

As important as the internal structural relations in the semantic domains are the relations which connect the various parts. As the reader will see from our introductory discussion of situation semantics, we have one basic relation showing the *interaction* of the various primitive parts:

(10) $$\mathit{in}\ s:\ \mathit{at}\ l: r, a_1, \ldots, a_n; \mathit{pol}$$

where *pol* is either 1 or 0. There may also be other interactions: we may want to have a relation which expresses that a location l occurs in a situation s, etc..

The direct mathematical study of such (many-sorted) algebraic structures seems a rewarding task, of which some indications are given at various places in Chapter IV. In the long run, it usually pays to have some "strategic depth," in studying one's ontology beyond the immediate needs of linguistic interpretation.

With this outline of our program, we now turn to the details of the specific contributions.

CHAPTER II

FROM LINGUISTIC FORM TO SITUATION SCHEMATA

In this section we connect the description of linguistic form (phonology, morphology and syntax) with situation schemata. We describe the relationship between meaning and linguistic form in terms of constraints. This differs from the more traditional approach where form, and in particular syntactic form, is related to interpretations through means of stepwise derivations. The constraint approach establishes the connection between form and meaning declaratively and without arbitrary conditions on the order of steps employed in actually linking form and content.

We first sketch the view we hold of semantic interpretation as the product of integration of constraints from a multiplicity of levels of description of linguistic form. Next we discuss the semantic representations —the situation schemata—which are the common denominator on the basis of which the determination of model-theoretic satisfaction conditions take place. Finally, we describe how the semantic representations and the linguistic form of the utterance are algorithmically related.

1 LEVELS OF LINGUISTIC FORM DETERMINING MEANING

The informational content of an utterance is determined by a number of contextual factors (the discourse situation, the resource situation etc.) and the linguistic form of the utterance, i.e. its phonology, morphology and syntax. An alternative way of putting the same point is to say that the different levels of linguistic form *constrain* the interpretation of the utterance. We want our interpretation system to reflect the semantic significance of all levels of linguistic form, not just syntax. The phonology of the utterance is clearly a determinant of its informational capacity. The tone of voice with which I utter "Oh, you have arrived" can tell you whether or not I am happy to see you. Similarly, the informational content in the utterance "No, Bill found Fido", changes drastically depending on what word, if any, carries emphatic stress. (Compare "No, Bíll found Fido" vs. "No, Bill found Fído"). Some of the uses of voice register and tone of voice are conventionalized and a proper part of the linguistic system. The last example of the use of emphatic stress is a case in point. This is something which properly falls within the limits of the theory of how language (considered as a symbolic system) carries information, and we will consider such phenomena in our analysis. In contrast, if you conclude that I am happy to see you from the tone of my voice when I say "Oh, you have arrived", you are probably relying on a non-conventionalized aspect of the use of tone of voice, or at least, it is a use whose conventions are not intimately tied up with the linguistic *structure* of the utterance. This type of phenomenon will not be of concern to us here.

Morphology (case endings, verb-markings for tense, aspect etc.) is similarly decisive for semantic interpretation. While word order has a pivotal status in English, many other languages assign that status to morphological information. Consider the Latin sentence *Scriba tabulam habet rasam.* The order of words in this sentence tells us little about its interpretation. It is the morphological endings on the words which make it possible for us to establish that the individual who has something is the scribe, (*scriba*, nominative case), and what he has is a tablet (*tabulam*, accusative case). Furthermore, it is the case ending on *rasam*, 'blank/clear', which tells us that this adjective modifies *tabulam*, rather than *scriba*.

The standard view in linguistics has been that syntactic structure is the (only) input to semantic interpretation. According to this view the organization of the components of a generative grammar can be summarized as in Figure 1.

In this model all information relevant for semantic interpretation must be channeled through the syntactic representation. Likewise for the phonology. It is driven off the syntax. The syntax is the only "generative" component; the phonology and the semantics are "interpretive" components.[1]

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[1]Cooper (1975) developed a version of "interpretive semantics" for transformational grammars where scope relations were not indicated in the syntax using what has later become known as "Cooper storage". It would be compatible with Cooper's theory to assume that the factors which influence the

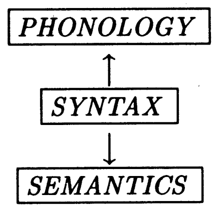


Figure 1: Derivation-based view of the relationship between linguistic form and meaning.

A particular version of this view holds the stronger position that the syntactic ***surface structure*** of the utterance is the input to semantic interpretation. The argument for this position goes as follows:

1. Syntactic surface structure is independently motivated (by syntactic arguments).
2. Syntactic surface structure provides the compositional structure necessary for semantic interpretation.
3. Therefore, the construction of other representations for the purpose of semantic interpretation is unnecessary.

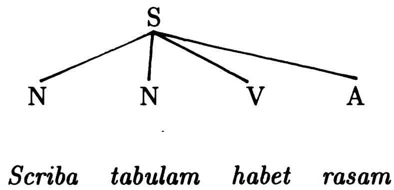
We are skeptical to this argument for two reasons. Most important is the obvious observation which we just made that a multiplicity of factors constrain the interpretation of utterances. This, we believe, should be reflected in the theory of interpretation. Another reason for rejecting the argument is that the second premise is in general not true. Free word-order languages, like Latin, Warlpiri, Hungarian etc., eloquently bear out the point that syntactically motivated surface constituent structure does not, in general, provide all the structure needed to guide semantic composition. The standard picture of semantic composition, which

---

selection of a preferred reading (eg. context and discourse structure) act directly on the quantification mechanism without this information being encoded in the syntactic form of the utterance, but this possibility was not explored at the time, and the readings generated by the semantics are all derivable directly by the semantic rules from the syntactic tree alone.

uses unary functions, requires a hierarchical control structure. No hierarchical surface structure is motivated by syntactic evidence for our simple Latin sentence ***Scriba tabulam habet rasam.*** The only unenhanced surface structure which is motivated with syntactic evidence for this sentence is the flat structure in (11) which is totally unrevealing with respect to the semantic relations holding in the sentence.

(11)



If attention is limited to artificially simple English sentences, one can be lead to the conclusion that there is an isomorphism between the syntactic surface structure of sentences, and what we can intuitively think of as their "logical syntax" (i.e. their compositional structure). But this reason for attaching special interest to the syntactic surface structure of utterances also disappears if we consider a more realistic fragment (eg. *There*-insertion constructions, sentences involving extraposition etc.). We do, of course, realize that this does not in any way mean that syntactic surface structure can not be successfully interpreted. One simply has to include among the operations available in the semantic component operations whose function it is to transform the surface constituent structure of the sentence to the logical syntax of the sentence.

We propose an analysis where syntax phonology and morphology constrain the semantics directly. All semantically relevant aspects of the linguistic form of the utterance as well as contextual factors contribute to a single pool of constraints which determine the meaning of the utterance (cf. Fig. 2). The constraint system is designed so as to leave the morphology, syntax and phonology untainted: We do not introduce semantic artifacts into other levels of analysis. We are striving towards a synthesis of well-motivated analyses of different linguistic levels. In our approach, all levels of linguistic description have equal theoretical status. They all stand in a mutually constraining relationship.There are principles internal to each level which determine the properties and well-formedness of the representation on that level. Each level has its

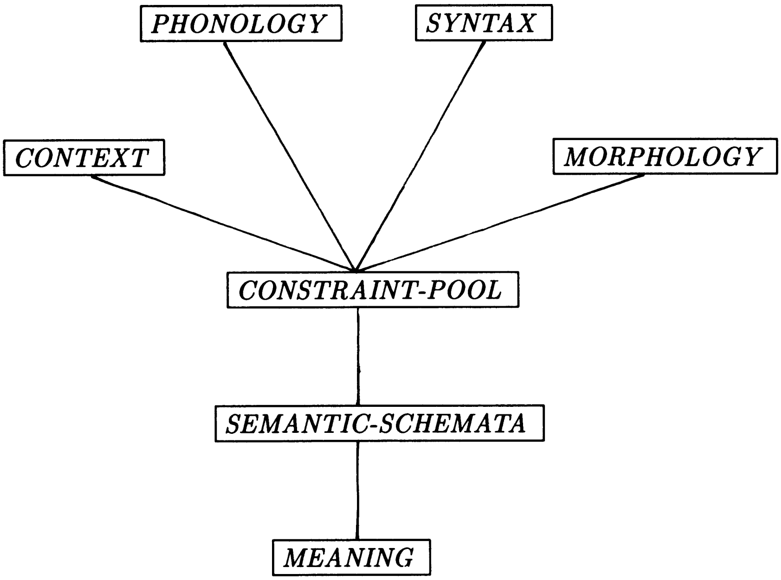


Figure 2: Constraint based view of the relationship between linguistic form and meaning

own set of generative rules. There is no asymmetry between interpretive and generative components.

The traditional view of the relationship between the semantics and levels of linguistic form is rooted in the notion of derivation in transformational grammar. In that theory, a derivation is a stepwise transformation of one representation into another. The transformations are productions which derive minimally different intermediate representations. The mapping is not a one-step mapping. The transformation of representations happens through a carefully ordered set of steps. This idiom naturally leads to the the view that the levels of representation are sequentially lined up as in figure 1. By viewing the relationship between form and meaning in terms of mutual constraints we can reflect more directly the intuition that the different aspects of linguistic form all partially determine the interpretation.

2 MOTIVATION FOR THE USE OF CONSTRAINTS

In the constraint based theory the focus is not on the derivation or construction of one level (say semantics) on the basis of another (typically syntax). The "inter-level" constraints, which we are examining here, are declarative descriptions of the relationships holding between aspects of linguistic form and the semantic representation itself which have to be satisfied if the attributed meaning is to correspond to the linguistic form. The constraints do not describe intermediate representations between linguistic form and semantic representation. (Though it is always possible to solve arbitrary subsets of equations, which gives us a handle on the treatment of partial information and the analysis of utterance fragments.)

Contrary to some linguistic practice, we do not want to adopt a notational system which in its syntax embeds a theory of translation and interpretation, i.e. a notation which only allows "true" statements to be made about our problem domain. We prefer to be conservative with respect to what assumptions we embed in the notational system and let the notation support exploration of alternatives until there is evidence in favor of specific choices. The constraint system provides us with such a general framework within which different translation strategies can be explored. Halvorsen (1983) used constraint equations in a system where the functional structures from lexical-functional grammar (Kaplan and Bresnan 1982) served as the representation of linguistic form and where the semantic target representation was higher order intensional logic (Montague 1970). Here our semantic target is situation schemata rather than intensional logic, and instead of having functional structures be the only input to semantic interpretation, functional information is here but one of several sources of constraints on the interpretation. The constraint system is equally compatible with either of the alternatives.

The constraints on interpretation emanate from the analysis of the linguistic form of the utterance and are declarative in nature: One example of a constraint is that if an utterance contains the phrase *kick Pluto* we know that somewhere in the semantic representation of the utterance there will be a relation *kick*, and its second argument will be an individual, *Pluto*. Another constraint would be that if the word *Pluto* receives contrastive stress in an utterance, then the semantic representation of the utterance is constrained to mark Pluto as the focus.

Following the practice of lexical-functional grammar we associate descriptions in the form of equations with each word and each constituent of the syntactic surface structure of the utterance. In LFG these structures are called functional structures, or f-structures. Our descriptions make explicit the semantic as well as the functional information, and we will refer to them using names such as d_i (for *description*), rather than f_i (for *functional-structure*).

The constraint that the situation schema for an utterance contain the relation *kick* is expressed in the equation (d_i *SITSCHEMA REL*) = *kick*. We read this as "d_i's *SITSCHEMA*'s *REL* is *kick*", where d_i stands for some description of the utterance, *SITSCHEMA* is an attribute in the description, and *REL* is in turn an attribute in the situation schema which forms part of this description. The constraint that the second argument of this same occurrence of *kick* be *Pluto* is stated as (d_i *SITSCHEMA ARG.2*) = *Pluto*. It may be the case that the only piece of information about d_i which we have is just what these equations express. But it is also possible that other equations have already constrained some of the properties of d_i. The different pieces of information are combined through *unification* (Robinson 1965; Kay 1979; Kaplan and Bresnan 1982).

The = sign in the constraining equations is an instruction to *unify* the value of the left-hand side of the equation with the value of the right-hand side of the equation (cf. Karttunen 1984). The effect of unification is different depending on the types of entities that are unified. Consider the equation (d_i *SITSCHEMA ARG.2*) = *Pluto*. In this case one side of the equation is an atomic element. In such cases the value of the left side is simply equated with the atomic element, i.e. the value of the attribute *ARG.2* in the semantic representation (*SITSCHEMA*) d_i is *Pluto*. The situation is different when there are feature name chains on both side of the = sign. If we have an equation (d_i *ARG.2*) = (d_j *ARG.1*) the result of unification is that the two attributes (*ARG.1* and *ARG.2*) *point to* one and the same value, here *Pluto*.

A situation schema for a given utterance is one which satisfies all the semantic constraints. (A semantic constraint is any conflict involving semantic attributes). There can be more than one semantic equation set for a given utterance. In such cases the utterance may be ambiguous.

However, ambiguity may also arise in the interpretation component even if there is a unique semantic schema for an utterance (cf. Chapter III).

Even though the constraints are normally taken to describe rather than derive the semantic representation it is also possible to construe the constraints in a more restricted fashion as instructions for how to construct the semantic representation. There is a construction algorithm for the constraint system we are working with here which, given a set of constraints in the form of equations, allows us to build, i.e. derive, the semantic representation (cf. II.4).

3 THE MODULARIZATION OF THE MAPPING FROM FORM TO MEANING

We divide the mapping between form and meaning into two modules which can be likened to the translation step and the interpretation step in Montague Grammar, though this analogy should not be taken too literally since the principles on which we divide tasks between the two modules are different. One component, which we describe in this chapter, relates semantic representations and representations of linguistic form (morphology, phonology, syntax and a functional analysis). The other component, which we describe in Chapter III, determines the satisfaction conditions for an utterance on the basis of its semantic representation(s). All the representations and rules involved in the first component set forth restrictions on the correspondence between semantic information an information about syntactic, phonological, functional and morphological structure of semantic significance. In the semantic representations which are the output of this component the informationally relevant aspects of the utterance are organized in ways which make it possible for the interpretation rules (discussed in Chapter III) to be fewer in number, simpler in nature, and less subject to cross-linguistic variation.

The interpretation rules in a *surface structure* based interpretation system for natural language perform two distinct types of functions: Meaning composition and recovery of a canonical order of operators, predicates and arguments. Consider the meaning which would be assigned to the active and the passive version of a verb such as *kick* in a Montagovian analysis.

(12) *kick* *kick'*
be kicked by $\lambda\mathcal{P}\lambda\mathcal{Q}kick({}^{\check{}}\mathcal{Q},{}^{\check{}}\mathcal{P})$

The translation assigned to the passive phrase *be kicked by*, has one component which is shared with its active equivalent (the logical constant *kick*). The other component (the enclosing lambda expressions), is a reflection of the fact that in English passives the agent argument to the verb is in post-verbal position, and part of the *VP*-constituent, rather than in the typical pre-verbal subject position, outside of the *VP*. In this paper we choose to split the two types of operations or rules apart. The establishment of the canonical predicate argument-relations is accomplished by the formal operations of the grammar (i.e. the operations that deal with the form of the utterance). In the semantic representation, i.e. the situation schema, the predicate-argument relations are recovered. The semantic operations (corresponding to functional application etc., in the Montague analysis) take place in our interpretation component.

The deviations from the canonical order of predicates and arguments differ radically from language to language. This is, of course, a source of syntactic disparity between languages. If the recovery of the canonical predicate-argument relations are left to the semantics, the consequence is that the interpretation rules will vary from language to language. Since the variable aspect of natural language is what has to be learned, the consequence of this approach is that the learnability of the semantic interpretation rules becomes an issue in much the same way as the syntactic rules of a language. One has to explicitly specify a theory of learning both for the semantic operations and for the syntax. Our intermediate level of semantic representation enables us to characterize the language particular aspects of the grammar, i.e. those that have to be learned as those components of the grammar that deal with form rather than content.[2]

[2]Notice that we are here talking about the grammar as distinct from the lexicon. Naturally, the lexical content of individual lexical items have to be learned.

4 SITUATION SCHEMATA

The conceptual and theoretical distinctions between levels of representation (such as syntactic, surface structure, functional representation and situation structure) is motivated on the basis of the explanatory power which can be gained from such a decomposition. The structure of our interpretation system has three modules:

(13)
$$\boxed{\begin{array}{l}\text{Levels of}\\ \text{linguistic form}\end{array}} \longleftrightarrow \boxed{\begin{array}{l}\text{Situation}\\ \text{Schemata}\end{array}} \longrightarrow \boxed{\begin{array}{l}\text{Model-theoretic}\\ \text{interpretation}\end{array}}$$

Trivially, given the uniform algorithmic mapping between the different levels of representation and its property of monotonicity, the different levels can all be collapsed into one comprehensive analysis for the utterance. In fact, this was done in one of the earliest implementations of the theory where functional representations were related to descriptions of sets of formulas of higher order intensional logic (Halvorsen 1983). From a theoretical point of view, this "integration" of levels has nothing to recommend it in the absence of positive empirical evidence.

Situation schemata are attribute-value pairs where the attributes name things which situations are analyzed in terms of, i.e. *REL*ation, *ARG*ument, *POL*arity, *LOC*ation, etc. The values are either atoms, which stand for relations, individuals and properties, or situation schemata.

The graph representation with attribute-value pairs, as in (14), is a favored representational format in a number of linguistic theories, specifically lexical-functional grammar and unification grammar.

(14)
$$\begin{bmatrix} \mathit{Attribute}_i & \mathit{Value}_i \\ \mathit{Attribute}_j & \mathit{Value}_j \\ \mathit{Attribute}_k & \mathit{Value}_k \end{bmatrix}$$

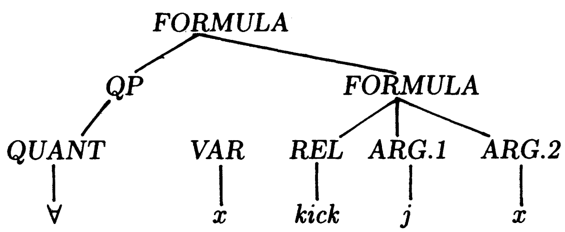
According to the view of semantic composition and interpretation which we are developing here, it is the semantic constraints that are of primary importance. They carry the information about how the different aspects of the linguistic form of the utterance constrain its interpretation. As we stated in Chapter I, a situation schema *summarizes* and makes explicit the information implicit in the collection of semantic constraints.[3] How the situation schema explicates the constraints on the interpretation can best be seen by considering the relationship between a situation schema and situation theoretic objects which satisfy the constraints which the situation schema summarizes.

Any formula can be rendered as an attribute-value graph given an analysis of the formula into its constituent parts. If we give the analysis of the formula as a phrase-structure tree the first level of nodes in the tree will correspond to the attributes at the top level of the graph. The subtrees dominated by the nodes of depth one define new attribute-value graphs which are the value of the attributes derived from the nodes of depth one. Consider the formula:

(15) $\forall x[kick(j,x)]$

We can analyze this formula as in (16):

(16)



The corresponding attribute-value graph is (17).

[3] Halvorsen (1987) develops this view in greater detail.

(17)
$$\begin{bmatrix} \text{FORMULA} & \begin{bmatrix} \text{QP} & \begin{bmatrix} \text{QUANT} & \forall \\ \text{VAR} & x \end{bmatrix} \\ \\ \text{FORMULA} & \begin{bmatrix} \mathit{REL} & \mathit{kick} \\ \mathit{ARG.1} & j \\ \mathit{ARG.2} & x \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

We sometimes simplify the graph by eliminating attribute labels corresponding to the root label (here "FORMULA"). These simplifications render the previous attribute-value graph as (18):

(18)
$$\begin{bmatrix} \text{QP} & \begin{bmatrix} \text{QUANT} & \forall \\ \text{VAR} & x \end{bmatrix} \\ \\ \mathit{REL} & \mathit{kick} \\ \mathit{ARG.1} & j \\ \mathit{ARG.2} & x \end{bmatrix}$$

The relationship between the attribute-value graph notation and the standard notation for situation-theoretic objects, such as *facts*, should now be obvious. Facts can be either located or unlocated. Barwise and Perry's standard way of representing facts is as in (19):

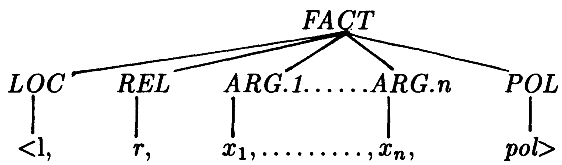
(19) *A located fact*:
$\textit{in } s : \textit{ at } l : r, a_1, \ldots, a_n; \textit{pol}$

Or more formally as a sequence such as (20):

(20) $< l, r, a_1 \ldots\ldots a_n, \textit{pol} >$

In this sequence l stands for a *location*, r stands for an n-adic *relation*, a_1 through a_n stand for the n arguments of the relation r, and *pol* represents a polarity. An unlocated fact simply has no location specification, l. With this knowledge of the *fields* of the notation of a fact we can construct the parse tree in (21) for the traditional fact-notation in (20):

(21)



The attribute-value version of the notation for the fact is therefore as in (22) where we have followed the convention of eliminating the attribute labels of the root-node (*FACT*).

(22)
$$\begin{bmatrix} LOC & \mathrm{l} \\ REL & \mathrm{r} \\ ARG.1 & a_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ ARG.n & a_n \\ POL & pol \end{bmatrix}$$

The fact in (19) had labels (variables) taking the place of locations, relations, and arguments. In (23) we have an unlocated fact with no unspecified roles, which expresses that 2 is greater than or equal to 1.

(23) $<\geq, 2, 1, 1>$

This fact is represented in our notation in (24).

(24)
$$\begin{bmatrix} \mathit{REL} & \geq \\ \mathit{ARG.1} & 2 \\ \mathit{ARG.2} & 1 \\ \mathit{POL} & 1 \end{bmatrix}$$

So far we have only considered atomic facts. This is a special case. Complex situation theoretic objects can also be assigned to the roles of the relation. Consider the situation type described in (25) which characterizes a situation in which John kicks Pluto at the time of utterance. Here the location is non-atomic. There is a restriction on the location that it spatio-temporally overlap the location of the utterance, l_d.

(25)
$$\begin{array}{l} \mathit{in}\ s : \text{at}\ l : \mathit{kick},\ \mathit{John}, \mathit{Pluto}, \mathit{1}; \\ \qquad\qquad l \circ l_d \end{array}$$

We handle this type of case in the attribute-value representation by introducing an *indeterminate* for the complex term, and then allowing conditions, or restrictions, to be associated with these indeterminates.[4]

(26)
$$\begin{bmatrix} \mathit{REL} & \mathit{kick} \\ \mathit{ARG.1} & \mathit{John} \\ \mathit{ARG.2} & \mathit{Pluto} \\ \mathit{LOC} & \begin{bmatrix} \mathit{IND} & \mathit{IND.1} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \circ \\ \mathit{ARG.1} & [\ ]_1 \\ \mathit{ARG.2} & l_d \end{bmatrix} \end{bmatrix} \\ \mathit{POL} & 1 \end{bmatrix}$$

---

[4]We use *indeterminate* as a label to mark a position or role in a situation schema. This usage may differ from the original sense intended in *Situations and Attitudes.*

The notation $[\ ]_i$ indicates a repeated reference to a shared attribute value, *IND.i*

IND is an attribute which can have indeterminates of different kinds as its value. Here its value, *IND.1*, is an indeterminate of the type of location. The attribute-value graph in (26) also illustrates the feature of our representations which make them graphs rather than trees. There is a link between the value of the *IND* of the *LOC* and the value of the *ARG.1* of the *COND* in the *LOC*. This indicates that this instance of *IND.1* has two parents. The *ARG.1* attribute and the *IND* attribute both point to one and the same instance of *IND.1*.

The set of well-formed situation schemata is completely specified by the rules in (27). The format of definition is meant to be entirely standard. There are a few abbreviations which we explain. REL^n signifies an n-ary relation constant. ARG^n stands for a sequence *ARG.1*,. . .,*ARG.n*, where each *ARG.i* can be further expanded. In the same way $SITSCHEMA^n$ abbreviates the sequence *SITSCHEMA.1*. . ., *SITSCHEMA.n*. Thus ($SITSCHEMA^n$) means that the occurrence of $SITSCHEMA^n$ is optional, i.e. in an expansion of an *ARG.i* there either is no occurrence of a *SITSCHEMA* or for some $n \geq 1$ there occurs a list *SITSCHEMA.1*,. . ., *SITSCHEMA.n*. *Entity* is exemplified by designators for individuals such as *John* and *Bill*. The reader should notice the special form of *COND* and $COND_{loc}$. The schemata corresponding to these attributes play a rather special role. A *COND* will constrain the domain of variation of a quantifier (see the remarks following (66) in III.1), and a $COND_{loc}$ will determine the location of a described situation (see the discussion of (60) in III.1). In order to prevent the rules in (27) from overgenerating, we have introduced a new type of argument attribute: *ARG′.i* in *COND* and $COND_{loc}$. The recursive force of the definition enters through the ($SITSCHEMA^n$) of an *ARG.i*.

(27)
$$
\begin{aligned}
&SITSCHEMA \rightarrow (SIT)(FOCUS)\ REL^n\ ARG^n\ LOC\ POL\\
&SIT \rightarrow \langle situation\ indeterminate\rangle\\
&FOCUS \rightarrow \{IND|IND_e\}\\
&REL_I \rightarrow \langle n\text{-}ary\ relation\ constant\rangle\\
&ARG.i \rightarrow \{\ IND_e|IND\ (SPEC)\ COND\ (SITSCHEMA)^n)\\
&\qquad\qquad (FOCUS)\}\\
&LOC \rightarrow IND\ COND_{loc}\\
&POL \rightarrow \{1|0\}\\
&IND_e \rightarrow \langle entity\rangle\\
&IND \rightarrow \langle indeterminate\rangle\\
&SPEC \rightarrow \langle quantifier\rangle\\
&COND \rightarrow (SIT)REL\ ARG'.1\ POL\\
&COND_{loc} \rightarrow REL_{loc}\ ARG'.1\ ARG'.2\\
&ARG'.i \rightarrow \{IND_e|IND\}
\end{aligned}
$$

As remarked in the introduction (see section I.1) the notion of situation schemata is open. The current version, (27), is adapted to the fragment that we present in section II.5. In Appendix A we shall see how to expand the *LOC* part of a situation schema to account for locative prepositional phrases. In the current version REL_{loc} will expand to the two relations $\circ$ (overlap) and $\prec$ (precede). To ensure the correct identification of roles we impose the further conditions

$$(ARG.i\ IND)=(ARG.i\ COND\ ARG'.1\ IND)$$

$$(LOC\ IND)=(LOC\ COND_{loc}\ ARG'.1\ IND)$$

To fix the "now" of the discourse location we assume that

$$(LOC\ COND_{loc}\ ARG'.2\ IND)=l_d$$

where l_d is an atomic value which in the interpretation of part 2 will be anchored to the discourse location.

The simplicity of the grammar for situation schemata is a one further reason for why we find it useful to work with this level of representation. The variety of form which our model-theoretic interpretation procedures have to cope with is all spelled out in the rules in (27).

The attribute-value representations of facts and situation types can be described by constraint equations. Consider the graph, (28), which represents the unlocated fact that 2 is greater than or equal to 1. Let us refer to this graph as d_1, which we indicate by annotating the box surrounding the graph with this identifier.

(28) $$d_1 \begin{bmatrix} \mathit{REL} & \geq \\ \mathit{ARG.1} & 2 \\ \mathit{ARG.2} & 1 \\ \mathit{POL} & 1 \end{bmatrix}$$

The *REL*ation attribute of d_1 is $\geq$. This is expressible by the constraint equation $(d_1\ \mathit{REL}) = \geq$. This equation makes the statement that there is a situation schema, d_1, and this situation schema has an attribute *REL* whose value is $\geq$. The first argument of this relation is the number 2, which we can express by stating that $(d_1\ \mathit{ARG.1}) = 2$. This equation states about d_1 that it has an attribute *ARG.1* (which comes in addition to *REL* whose existence was asserted by the previous equation), and the value of this attribute is the number 2. Similarly for the second argument of the relation we have $(d_1\ \mathit{ARG.2}) = 1$ which claims the existence of yet another attribute, *ARG.2* with the value 1. And the information about the polarity of the attribute-value pair in (28) is encoded in the equation $(d_1\ \mathit{POL}) = 1$ which claims that there is an attribute *POL* in d_1 with the value 1.

(28) is the smallest situation schema which satisfies the description given by the constraint equations. There are larger situation schemata which mention other attributes which also are compatible with the constraints. In particular, the situation schema which has the attribute *ARG.3* with the value *Mary* in addition to the four attribute-value pairs which (28) has, also satisfies the constraints.[5] If we were to start out

[5]One might think that this expanded situation schema ought to be ruled out because $\geq$ is a dyadic relation, while the additional equation asserts the existence of a third argument, but we consider this a separate task.

with the equations rather than the situation schema which they describe we could construct the schema in the following manner. When faced with the equation $(\alpha\ \beta) = \gamma$ we construct a place-holder for a situation schema since this type of equation asserts the existence of a schema α.

(29) $\quad {}_{\alpha}[\qquad\quad]$

In fact we can add more structure to the schema we are building since the expression $(\alpha\ \beta)$ on the left-hand side of the = sign tells us that α must have an attribute β. This gives rise to (30) where the underline signals that the value of the attribute is not yet known.

(30) $\quad {}_{\alpha}[\beta\ \ \underline{\qquad}]$

The appearance of γ on the right-hand side of the = sign licenses the replacement of the underline with γ as in (31).

(31) $\quad {}_{\alpha}[\beta\quad \gamma]$

When another equation which makes mention of α is encountered, we simply add structure to the already existing situation schema. In particular, the constraint-equation $(\alpha\quad \xi) = +$ would license the expansion of (31) to (32):

(32) $$\alpha\begin{bmatrix}\beta & \gamma\\ \xi & +\end{bmatrix}$$

If ξ is also associated with another atomic value, as through the equation $(\alpha\ \xi) = -$, an inconsistency arises, and we know that the equation set can not have a solution. We will consider the construction algorithm for situation schemata in greater detailed in connection with the building up of a situation schema for a complex sentence in the next section.

5 THE ALGORITHM FROM LINGUISTIC FORM TO SITUATION SCHEMATA

Now that we have precisely defined what a situation schema is and how situation schemata can be described by means of constraint equations, it is time to turn to the question of how situation schemata and other representations of linguistic form can be algorithmically connected.

Recall that one of our main concerns is to construct a system where all levels of representation of linguistic form directly constrain the situation schema and thus the interpretation of the utterance. Another goal is to produce a system where all levels are created equal, i.e. where there is no distinction between levels in their importance for interpretation, nor is there any a priori distinction between "generative" and "interpretive" levels. The design which meets these criteria is one where the mutually constraining relationship of the levels of linguistic representation is mediated by a monotonic system of constraint equations, and where all levels of linguistic form introduce constraint equations. We are also interested in creating a system which can be smoothly integrated with research in other linguistic disciplines than semantics, specifically syntax, phonology and morphology. To illustrate how we think this can be done we adopt as our syntactic component a lexical-functional grammar (cf. Kaplan and Bresnan 1982, Bresnan 1982). In fact, our strategy for relating linguistic form and semantic schemata borrows heavily from lexical-functional grammar in that it is an adaptation of the system for relating word-strings to functional structures in that theory. The difference is that we produce representations especially tailored for interpretation in a situation semantics, whereas the functional structures produced by LFG are designed to support explanations of syntactic phenomena. We will not be concerned if there are cases where the syntactic account appears to overlap with our system for relating linguistic form and meaning. The semantic schemata do not presuppose a functional analysis as presented in functional structures, even though it does facilitate the task of producing the semantic schemata. As a phonological and morphological base one could assume a version of the lexical theory of morphology and phonology (Kiparsky 1982). The functional, semantic, phonological and morphological levels of analysis can all be described by constraining equations.[6] It is only the surface phrase-structure tree which fails to

[6]Cf. Withgott and Halvorsen (1984) for a morphological and phonological analysis using constraining equations

be generated in this manner. This is an inelegant asymmetry which we hope can be avoided in the future.[7]

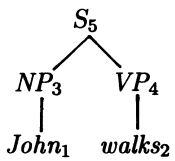
The details of these theories are irrelevant for our present purposes, and the approach to translation and interpretation which we are presenting here does not depend on the adoption of any particular syntactic, morphological or phonological theory. We shall therefore talk about the phonological, morphological and syntactic representations only as required.

Consider the simple sentence in (33) where the words are indexed with subscripts so that correspondences between the word string and associated descriptions in terms of attribute-value graphs can be more easily stated.

(33) $John_1$ $walks_2$

This sentence can be associated with the constituent-structure tree in (34), and a description whose functional aspect is the functional structure in (35).

(34)



(35)

$$\begin{array}{l} f_2\\ f_4 \\ f_5 \\ \\ \\ \end{array}\left[\begin{array}{ll} \mathit{SUBJ} & \begin{array}{l}f_3\\f_1\end{array}\left[\begin{array}{ll} \mathit{PRED} & \text{'John'}\\ \mathit{NUM} & \text{SG}\end{array}\right] \\ & \\ \mathit{TENSE} & \text{PRESENT} \\ \mathit{PRED} & \text{'walk<}[\ ]_{f_1,f_3}\ \text{>'}\end{array}\right]$$

The association between sentence, surface-constituent structure and functional structure is accomplished by a grammar consisting of context-free rewriting rules with functional annotations in the form of equations.

---

[7]Cf. Chomsky (1982) where phrase-structure is epiphenomenal.

The grammar in (36) jointly with the lexicon in (37) would assign the structures in (34) and (35) to the sentence in (33).[8]

(36)
$$\begin{array}{llll} S & \rightarrow & NP & VP \\ & & (\uparrow SUBJ) = \downarrow & \uparrow=\downarrow \\ VP & \rightarrow & V & \\ & & \uparrow=\downarrow & \end{array}$$

(37)
$$\begin{array}{llll} John & NP & * & (\uparrow PRED) = \downarrow \\ & & & (\uparrow NUM) = SG \\ walk & V & \text{-}s & (\uparrow PRED) = 'walk < (\uparrow SUBJ) >' \end{array}$$

When a phrase structure rule is used in the construction of a phrase structure tree, the functional descriptions (i.e. the constraint equations which describe the functional attributes of the phrase) associated with that rule are contributed to the pool of constraints which describe the functional structure of the sentence on the analysis which the phrase-structure tree determines. In (38) the phrase-structure nodes are annotated to show the individual equations associated with the rules used to generate each node in the tree.

(38)
$$\begin{array}{ccc} & S_5 & \\ (\uparrow SUBJ) = \downarrow & & \uparrow=\downarrow \\ NP_3 & & VP_4 \\ | & & | \\ (\uparrow PRED) = 'John' & & \\ (\uparrow NUM) = SG & & (\uparrow PRED) = 'walk <\uparrow SUBJ >' \\ \uparrow=\downarrow & & \uparrow=\downarrow \\ | & & | \\ John_1 & & walks_2 \end{array}$$

In order for these descriptions to determine an attribute-value graph, the meta-variables $\uparrow$ and $\downarrow$ have to be *instantiated*. With the meta-variables present we know that the equation $(\uparrow NUM) = SG$ makes

[8]The * in the lexical entry for *John* indicates that this word takes no affixes.

the claim that some attribute-value graph (referred to by $\uparrow$) has the attribute *NUM* with the value *SG*. And the equation $(\uparrow\ SUBJ) = \downarrow$ makes the claim that some attribute-value graph (again referred to by $\uparrow$) has the attribute *SUBJ*, and that this attribute has some attribute-value graph (referred to by $\downarrow$) as its value. But these descriptions are not complete until we know the identity of the attribute-value graphs which the equations make reference to. In this particular example we need to find out (*a*) whether the attribute-value graph with the attribute *SUBJ* and the attribute-value graph with the attribute *NUM* which the equations refer to are one and the same graph or different graphs, and (*b*) we need to determine the identity of the attribute-value graph which is claimed to be the value of the attribute *SUBJ*. The key to the instantiation of the meta-variables is simply that $\uparrow$ refers to the functional attribute-value graph of the description of the node dominating the current node. $\downarrow$ refers to the functional attribute-value graph of the description of the node that the equation attaches to. (39) is a modification of (38) where the metavariables have been instantiated. Here, we let d_i refer to the functional attribute-value graph of the description of the node indexed by i.[9]

(39)

S_5

$(d_5\ SUBJ) = d_3$ $\quad$ $(d_5 = d_4)$

NP_3 $\quad$ VP_4

$(d_3\ PRED) = {'John'}$

$(d_3\ NUM) = SG$ $\quad$ $(d_4\ PRED) = {'walk} < d_4 SUBJ > {'}$

$d_1 = d_3$ $\quad$ $d_2 = d_4$

$John_1$ $\quad$ $walks_2$

The complete set of constraint equations scattered through the tree in (39) are collected in (40).

[9]The constraint equations immediately over the lexical items ($d_1 = d_3$ and $d_2 = d_4$) are not specified in the lexical entries for the individual words. They are reflexes of an equation, $\uparrow = \downarrow$, which is associated with every lexical category through a redundancy rule. The lexical categories are often not present in the tree displays for space reasons.

(40)
$$\begin{aligned}
&d_1 = d_3\\
&d_2 = d_4\\
&(d_3\ NUM) = SG\\
&(d_3\ PRED) = {}'John'\\
&(d_4\ PRED) = {}'walk < (d_4\ SUBJ) > '\\
&(d_5\ SUBJ) = d_3\\
&d_5 = d_4
\end{aligned}$$

These are the constraints that determine the functional attribute-value graph in (35). Through unification we can find value assignments to the variables (d_i) in the equations which will satisfy them all. These value assignments are reflected in the subscripts on the functional structure in (35): d_2, d_4 and d_5 have assigned as their value the top-level attribute-value graph in (35). Similarly, d_3 and d_1 are bound to the attribute-value graph which is the value of the *SUBJ* attribute.

The functional structure is but one aspect of the description of the utterance. It is set up to support accounts of syntactic phenomena. We can also concentrate on the semantically significant information in the utterance and construct another part of the description of the utterance: the situation schema. As is the case for functional representations, some of the constraint equations that describe the situation schema of an utterance are introduced with the lexical entries for the morphemes which are used in the utterance. Others originate with the phonological analysis, yet others arise form the syntactic structure.

A situation schema of *John walks* is given in (41).

(41)
$$\begin{bmatrix} \mathit{REL} & \mathit{walk} \\ \mathit{ARG.1} & \begin{bmatrix} \mathit{IND}_e & \mathit{John} \end{bmatrix} \\ \mathit{LOC} & \begin{bmatrix} \mathit{IND} & \mathit{IND.1} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \circ \\ \mathit{ARG'.1} & [\,]_1 \\ \mathit{ARG'.2} & l_d \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

In (42) we have supplied the three morphemes *John*, *walk*, and *-s* with semantic descriptions. We will later see how the functional and the semantic descriptions are combined, but for right now we will concentrate on the semantic aspects of the description of the utterance.

(42)
$$\begin{array}{llll} \mathit{John} & \mathit{NP} & * & (\uparrow\ \mathit{IND}_e) = \mathit{John} \\ \mathit{walk} & \mathit{V} & \text{-}s & (\uparrow\ \mathit{ARG.1}) = (\uparrow\ \mathit{SUBJ}) \\ & & & (\uparrow\ \mathit{REL}) = \mathit{walk} \\ \text{-}s & & & (\uparrow \mathit{LOC}) = \downarrow \\ & & & (\downarrow\ \mathit{IND}) = \mathit{IND}.\alpha \\ & & & (\downarrow\ \mathit{COND\ REL}) = \circ \\ & & & (\downarrow\ \mathit{COND\ ARG'.1}) = (\downarrow\ \mathit{IND}) \\ & & & (\downarrow\ \mathit{COND\ ARG'.2}) = l_d \\ & & & (\uparrow\ \mathit{POL}) = 1 \end{array}$$

Given this information we can construct the tree in (43), where the constraining equations describe a situation schema rather than a functional structure. [10]

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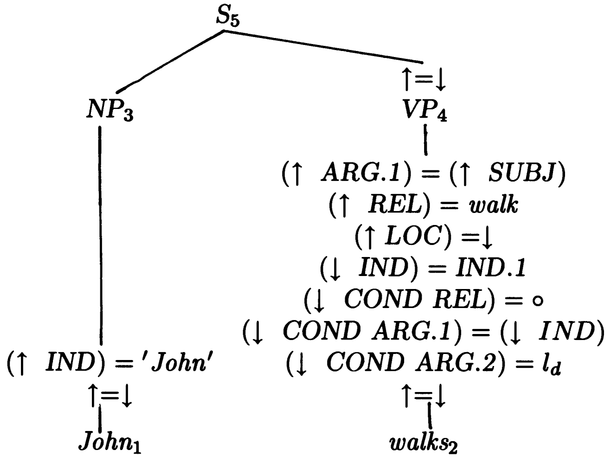
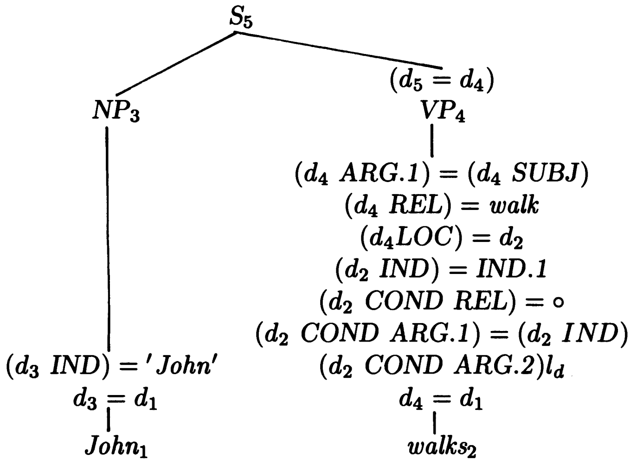
[10] In the following we frequently drop the prime-mark $'$ on the *ARG* of *COND* and COND_{loc}, and the subscripts on IND_e and COND_{loc}. These diacritics were introduced to limit the generative capacity of the grammar for situation schemata, but they are not of importance in the interpretation step.

(43)



Once again we can instantiate.

(44)



In (45) we have collected all the relevant equations for the sentence *John walks*, which are scattered throughout the tree in (44).

(45) $(d_5\ SUBJ) = d_3$
$d_5 = d_4$
$(d_3\ IND) = 'John'$
$(d_4\ ARG.1) = (d_4\ SUBJ)$
$(d_4\ REL) = walk$
$(d_4 LOC) = d_2$
$(d_2\ IND) = IND.1$
$(d_2\ COND\ REL) = \circ$
$(d_2\ COND\ ARG.1) = (d_2\ IND)$
$(d_2\ COND\ ARG.2) = l_d$
$d_3 = d_1$
$d_4 = d_1$

The equations in (45) also show the syntactic and semantic information interacting. It is the equation $(\uparrow\ SUBJ) = (\uparrow\ ARG.1)$ which is crucial in this connection. The straightforward interpretation of this equation is an instruction to unify the functional structure which is the value of $(\uparrow\ SUBJ)$ with the semantic schema which is the value of $(\uparrow\ ARG.1)$. However, this is not the intended interpretation. Instead, this equation signals that the semantic schema which is the value of $(\uparrow\ ARG.1)$ should be unified with the semantic schema which is associated with the description which the $\uparrow$ of $(\uparrow\ SUBJ)$ refers to. In other words, *ARG.1* should be unified with the semantic information pertaining to the syntactic subject. Which parts of the phrase make up the subject is determined by the *functional* descriptions.

This discussion implies a structuring of the grammatical information and the various types of equations which is more detailed than we have explicitly acknowledged. In order for the different components of the linguistic description to communicate properly we postulate that the attribute-value graphs, such as the functional structure and the situation schema of an utterance, are all included in a more comprehensive graph description which gives the complete analysis of the entire utterance.[11] In other words, if α is a functional

[11]Unification Grammar (Kay 1979) proposes an attribute-value graph representation for linguistic analysis where all aspects of the information re-

attribute in a non-mixed equation, then $(d_i\ \alpha)$ is an abbreviation for $(d_i\ \mathit{FSTRUC}\ \alpha)$. And if β is a semantic attribute, then $(d_i\ \beta)$ is an abbreviation for $(d_i\ \mathit{SITSCHEMA}\ \beta)$. If we have a mixed equation, as in $(d_i\ \mathit{ARG.1}) = (d_i\ \mathit{SUBJ})$, this is in fact short-hand for $(d_i\ \mathit{SITSCHEMA}\ \mathit{ARG.1}) = (d_i\ \mathit{FSTRUC}\ \mathit{SUBJ}\ \mathit{SITSCHEMA})$. This means that, on the top level, the analysis of the sentence *John walks* will look something like (46).

(46)
$$d_5\begin{bmatrix} \mathit{SITSCHEMA} & \begin{bmatrix} \mathit{REL} & \mathit{walk} \\ \mathit{ARG.1} & \begin{bmatrix} \mathit{IND} & \mathit{John} \end{bmatrix} \\ \mathit{LOC} & \begin{bmatrix} \mathit{IND} & \mathit{IND.1} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \circ \\ \mathit{ARG.1} & [\ ]_1 \\ \mathit{ARG.2} & l_d \end{bmatrix} \end{bmatrix} \\ \mathit{POL} & 1 \end{bmatrix} \\ \mathit{FSTRUC} & \begin{matrix} d_2 \\ d_4 \\ d_5 \end{matrix}\begin{bmatrix} \mathit{SUBJ} & d_3\begin{bmatrix} \mathit{PRED} & \text{'John'} \\ \mathit{NUM} & \text{SG} \end{bmatrix} \\ \mathit{TENSE} & \text{PRESENT} \\ \mathit{PRED} & \text{'walk<}[\ ]_{d3}\quad\text{>'} \end{bmatrix} \end{bmatrix}$$

We refer to the attribute-value graph which contains both a situation schema and a functional structure as the *complete* description of the utterance. Usually, we do not display both aspects of the analysis for

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garding the utterance can be comprehensively represented. We go beyond Kay's work in providing a detailed account of the semantics and the syntax/semantics interface.

Subsequent to the appearance of Fenstad et al (1985), a representational format and an interpretation strategy similar to ours has been developed around head-driven phrase structure grammars (HPSG) by Pollard and Sag (forthcoming).

space reasons, and when we need to refer to the connection between the semantic descriptions and the functional, or other, descriptions of linguistic form, we present both the functional structure and the situation schema for only the relevant parts of the utterance (cf. (46) where it is only on the top level that both the functional structure and the semantic schema of the complete description are displayed). However, it should be kept in mind that whenever a functional structure is encountered, there is a corresponding (possibly empty) semantic schema, and vice versa.

We will now look in more detail at how the functional and semantic information is communicated as the description for the utterance *John walks* is built up. When we combine the functional and semantic descriptions associated with the *NP* node (cf. (39) and (44))) we find that the complete description of this phrase is (47).

(47)
$$\begin{bmatrix} \mathit{SITSCHEMA} & \begin{bmatrix} \mathit{IND} & \mathit{John} \end{bmatrix} \\ \mathit{FSTRUC} & \begin{bmatrix} \mathit{PRED} & \text{`John'} \\ \mathit{NUM} & \text{SG} \end{bmatrix} \end{bmatrix}$$

If we take one more step up the phrase-structure tree for *John walks*, we encounter the functional description, $(d_5 \;\; SUBJ) = d_3$. This licenses the introduction of a new level of f-structure relating to the top node in the phrase-structure tree, S_5. The only attribute which we have knowledge of on this level as of yet is the *SUBJ* attribute mentioned in the equation. The complete description of the utterance is now as in (48).[12]

[12]Notice that "the complete description" does not refer to a description with all the information in the utterance in it, but rather to an attribute-value graph with both an *FSTRUC*-attribute and a *SITSCHEMA*-attribute. Each of these descriptions may contain only partial information about the phrase.

(48) $$d_5 \left[\begin{array}{ll} \mathit{FSTRUC} & \left[\begin{array}{ll} \mathit{SUBJ}\ d_3 & \left[\begin{array}{ll} \mathit{SITSCHEMA} & \left[\begin{array}{ll} \mathit{IND} & \mathit{John} \end{array}\right] \\ \mathit{FSTRUC} & \left[\begin{array}{ll} \mathit{PRED} & \text{'John'} \\ \mathit{NUM} & \mathit{SG} \end{array}\right] \end{array}\right] \end{array}\right] \end{array}\right]$$

There is no semantic attribute on the top level of this complete description yet. This is a reflection of the fact that at this point we have only partial information about the semantic structure of the utterance. In particular, we do not know how the interpretation of the initial *NP* fits in with the interpretation of the rest of the sentence. This information is only available when more of the equations pertaining to the verb-phrase of the utterance have been analyzed. It is in the verb-phrase that we find the equation which was the starting point for this long excursion into the communication between syntax and semantics, $(d_4\ \mathit{ARG.1}) = (d_4\ \mathit{SUBJ})$. The left-hand side of this equation, $(d_4\ \mathit{ARG.1})$, and the equation $d_4 = d_5$, tells us that the complete description d_5, has a semantic schema with the attribute *ARG.1*. The value of *ARG.1* is the semantic schema of the complete description which is the value of (d_5 FSTRUC SUBJ) in (48). We now have enough information about the utterance to construct the attribute-value graph in (49).

(49) $$d_5 \left[\begin{array}{ll} \mathit{FSTRUC} & \left[\begin{array}{ll} \mathit{SUBJ} & \left[\begin{array}{ll} \mathit{SITSCHEMA} & \left[\begin{array}{ll} \mathit{IND} & \mathit{John} \end{array}\right] \\ \mathit{FSTRUC} & \left[\begin{array}{ll} \mathit{PRED} & \text{'John'} \\ \mathit{NUM} & \text{SG} \end{array}\right] \end{array}\right] \end{array}\right] \\ \mathit{SITSCHEMA} & \left[\begin{array}{ll} \mathit{ARG.1} & \left[\begin{array}{ll} \mathit{IND} & [\,]_{\mathit{John}} \end{array}\right] \end{array}\right] \end{array}\right]$$

Now that we know the correct role of the semantic information pertaining to *John* in the utterance as a whole, i. e. it is the *ARG.1*, of the main relation of the utterance, we will suppress the display of the semantic schema inside the *SUBJ* f-structure.

(50)
$$d_5 \begin{bmatrix} \mathit{FSTRUC} & \begin{bmatrix} \mathit{SUBJ} & \begin{bmatrix} \mathit{PRED} & \text{'John'} \\ \mathit{NUM} & \text{SG} \end{bmatrix} \\ & \end{bmatrix} \\ & \\ \mathit{SITSCHEMA} & \begin{bmatrix} \mathit{IND} & \mathit{John} \end{bmatrix} \end{bmatrix}$$

The remainder of the constraining-equations in (45), both the functional and the semantic, are of the standard homogeneous variety (i.e. they do not contain a mix of functional and semantic attributes). Straightforward processing of these leaves us with (46), presented earlier, as the complete description of *John walks*. In (46) the pointers between the embedded descriptions of the different aspects of the utterance are once again suppressed.

We have now illustrated a framework for linguistic description where semantic and syntactic information is propagated by means of constraining equations and where the syntactic and semantic information concerning an utterance is represented as distinct, but linked, attribute-value graphs. Another of our design requirements, which we set forth in the beginning of this chapter, concerned the integration of constraints on interpretation from other aspects of the formal analysis of the utterance than the syntax. The first step towards the integration of phonological information into the complete description of the utterance is the augmentation of the lexicon to include phonological information. For the proper noun *John* we can imagine a lexical entry as in (51).

(51) *John* *NP* * *Functional Information:* $(\uparrow \mathit{PRED}) = \text{'}\mathit{John}\text{'}$
$(\uparrow \mathit{NUM}) = \mathit{SG}$
Semantic Information: $(\uparrow \mathit{IND}) = \mathit{John}$
Phonological: $(\uparrow \mathit{PHONFORM}) = /\check{j}an/$
Information

An example of non-syntactic information which constrains the interpretation of an utterance is phonological stress. Consider a situation where our example sentence is uttered with main sentence stress on the subject, rather than on the final verb, as in *Jóhn walks*. This way of uttering the sentence would signal that *John* is the *FOCUS* of the utterance, and this has consequences for the interpretation of the sentence and the discourse which it is part of. In the phonological analysis of the utterance the contrastive stress on *John* would license the introduction of an equation ($\uparrow$ *CSTRESS*) $=\downarrow$.[13] The annotations on the phrase-structure tree associated with *John* should therefore be augmented as shown in (52).

(52)
$$\begin{array}{c}
(\uparrow\ \mathit{SUBJ}) = \downarrow \\
\mathit{NP}_3 \\
| \\
(\uparrow\ \mathit{PRED}) = \text{`John'} \\
(\uparrow\ \mathit{NUM}) = \mathit{SG} \\
(\uparrow\ \mathit{IND}) = \mathit{John} \\
(\uparrow\ \mathit{CSTRESS}) = \downarrow \\
\uparrow = \downarrow \\
| \\
(\uparrow\ \mathit{PHONFORM}) = /\check{j}an/ \\
\mathit{J\acute{o}hn}_1
\end{array}$$

As before the interpretation of $\uparrow$ and $\downarrow$ is context dependent. When $\uparrow$ precedes a phonological attribute name it refers to the phonological description of the dominating node. When $\downarrow$ occurs next to a phonological attribute name it refers to the phonological aspects of the description of the current node. With instantiation of the meta-variables, the equations in (52) will yield a set of equations which describe the attribute-value graph in (53).

(53)
$$d_5 \begin{bmatrix} \mathit{FSTRUC} & \begin{bmatrix} \mathit{SUBJ} & \begin{bmatrix} \mathit{PRED} & \text{`John'} \\ \mathit{NUM} & \mathrm{SG} \end{bmatrix} \end{bmatrix} \\ \mathit{SITSCHEMA} & \begin{bmatrix} \mathit{IND} & \mathit{John} \end{bmatrix} \\ \mathit{PHONSTRUC} & \begin{bmatrix} \mathit{PHONFORM} & /\check{j}an/ \\ \mathit{CSTRESS} & [\,]_{\check{j}an} \end{bmatrix} \end{bmatrix}$$

[13]Stress being a relative notion, the decision that *John* bears the main stress in the utterance can of course not be decided locally. The other pieces of information pertaining to the phrase are locally inferable.

Now that we have captured the phonological fact that the word *John* is the CSTRESS, how do we transmit this fact to the semantic analysis? Let us assume that a phrase which carries contrastive stress always has the *FOCUS* discourse function. On this assumption we can defaultly introduce the equation ($\uparrow$ *FOCUS*) = ($\uparrow$ *CSTRESS*) whenever the equation ($\uparrow$ *CSTRESS*) $=\downarrow$ is introduced. This will have the effect that the semantic description which is linked to the phonological phrase carrying the contrastive stress will be the value of the semantic *FOCUS* attribute in the semantic schema for any phrase containing the contrastive stress. In particular, for *Jóhn* in the utterance under consideration, we would have the complete description graph in (54).

(54)
$$d_5 \begin{bmatrix} \mathit{FSTRUC} & \begin{bmatrix} \mathit{SUBJ} & \begin{bmatrix} \mathit{PRED} & \text{`John'} \\ \mathit{NUM} & \text{SG} \end{bmatrix} \end{bmatrix} \\ & \\ \mathit{SITSCHEMA} & \begin{bmatrix} \mathit{IND} & \mathit{John} \\ \mathit{FOCUS} & \begin{bmatrix} \mathit{IND} & [\,]_{\mathit{John}} \end{bmatrix} \end{bmatrix} \\ & \\ \mathit{PHONSTRUC} & \begin{bmatrix} \mathit{PHONFORM} & /\check{j}an/ \\ \mathit{CSTRESS} & /[\,]_{\check{j}an}/ \end{bmatrix} \end{bmatrix}$$

We locate the value of *FOCUS* by tracing pointers from the phonological information to the semantic information corresponding to focused elements in the same way as we traced the pointer from the syntactic information to the semantic information while locating the *ARG.1* attribute.

The information that *Jóhn* is the *FOCUS* of the utterance can be propagated up from the *NP* -level to *S*-level of the analysis if that is desired. This would require the introduction of equations of the form ($\uparrow$ *FOCUS*) = ($\downarrow$ *FOCUS*).[14]

[14]The constraints on the distribution of foci in an utterance are complicated. The introduction of the equations which propagate the focus information must obey these constraints. If an utterance could have at most one focus the distribution of the propagation equations would be trivial: They would

When we integrate the information concerning the phonological stress and the focus in the complete description of *John walks*, we get a representation along the lines of (55).

(55)

$$
d_5 \left[\begin{array}{ll}
\mathit{SITSCHEMA} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{walk} \\
\mathit{ARG.1} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{John} \\
\mathit{FOCUS} & \left[\begin{array}{ll} \mathit{IND} & [\;]_{John} \end{array}\right]
\end{array}\right] \\
\mathit{LOC} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.1} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \circ \\
\mathit{ARG.1} & [\;]_1 \\
\mathit{ARG.2} & l_d
\end{array}\right]
\end{array}\right] \\
\mathit{FOCUS} & \left[\begin{array}{ll} \mathit{IND} & [\;]_{John} \end{array}\right] \\
\mathit{POL} & 1
\end{array}\right] \\
\mathit{FSTRUC} & \begin{array}{l} d_2 \\ d_4 \\ d_5 \end{array} \left[\begin{array}{ll}
\mathit{SUBJ} & d_3 \left[\begin{array}{ll}
\mathit{PRED} & \text{'John'} \\
\mathit{NUM} & \text{SG}
\end{array}\right] \\
\mathit{TENSE} & \text{PRESENT} \\
\mathit{PRED} & \text{'walk}< [\;]_{d3} \quad >\text{'}
\end{array}\right]
\end{array}\right]
$$

This treatment of the constraining effect of phonological information on interpretation is extremely sketchy. But the purpose of the discussion of the relationship between phonological and semantic information is only to provide an example of one way in which these aspects of the information can be integrated using the present system.

In the next chapter we provide a detailed account of how situation schemata for a wide variety of English utterances can be assigned interpretations in a situation semantics. To make good on our claim that the framework we are presenting helps integrate existing approaches to the

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occur on the node with the ($\uparrow$ *FOCUS*) $=\downarrow$ equation and on every dominating node. However, the truth is most likely more complicated.

analysis of linguistic form with a well-founded semantics, we now present a grammar characterizing the phrase-structure trees, functional structures and situation schemata for all the utterances which are discussed in detail in the remainder of the book. We have also built a sentence analysis system around this grammar and the "LFG Grammar Writer's Workbench" developed at Xerox PARC (Halvorsen 1987).[15]

(56) *The syntactic rules, functional and semantic equations*

$$\text{NP} \rightarrow \left\{ \begin{array}{l} \text{(DET) N} \\ \text{NPROP} \\ \text{NP } \bar{\text{S}} \\ \quad (\uparrow \text{ SS IND}) = (\downarrow \text{ FSTRUC TOPIC SS}) \\ \quad \downarrow \in (\uparrow \text{ FSTRUC RELMOD}) \\ \quad (\downarrow \text{ SS}) \in (\uparrow \text{ SS SITSCHEMA}) \end{array} \right\}$$

$$\text{S} \rightarrow \left(\begin{array}{l} \text{NP} \\ (\uparrow \text{ FSTRUC SUBJ}) = \downarrow \end{array} \right) \text{ (AUX) } \begin{array}{l} \text{VP} \\ \left\{ \begin{array}{l} \uparrow = \downarrow \\ (\uparrow \text{ FSTRUC VCOMP}) = \downarrow \end{array} \right\} \end{array}$$

$$\bar{\text{S}} \rightarrow \begin{array}{l} \text{NP} \\ (\uparrow \text{ FSTRUC TOPIC}) = \downarrow \\ \left(\uparrow \text{ FSTRUC} \left\{ \begin{array}{l} \text{COMP FSTRUC} \\ \text{VCOMP FSTRUC} \end{array} \right\}^{*} \left\{ \begin{array}{l} \text{SUBJ} \\ \text{OBJ} \end{array} \right\} \right) = \downarrow \end{array} \quad \text{S}$$

$$\text{VP} \rightarrow \text{V} \left(\begin{array}{l} \text{NP} \\ (\uparrow \text{ FSTRUC OBJ}) = \downarrow \end{array} \right)$$

[15]In the fragment that follows an $\uparrow=\downarrow$ equation is implied if no other equations appear underneath a category symbol in the right-hand side of a rule. The symbol, @, signifies a form with irregular morphology. Long-distance dependencies are handled using the regular expression capability of the description language for LFG and the notion of functional uncertainty, which permits infinite classes of constraints to be specified and analyzed (Kaplan et al forthcoming).

In the fragment SS is used for SITSCHEMA on the top-level of complete descriptions, i.e. when cooccurring with the FSTRUC attribute, in order to distinguish this use of the attribute from its use in introducing restrictions on arguments (cf. the NP $\bar{\text{S}}$ expansion of the NP-rule and the defintions in (27)).

-NSG	AFF	*
		(↑ FSTRUC PERSON)=3
		(↑ FSTRUC NUM)=SG
-V3SG	AFF	*
		(↑ FSTRUC TENSE)=PRESENT
		(↑ FSTRUC SUBJ FSTRUC NUM)=SG
		(↑ FSTRUC SUBJ FSTRUC PERSON)=3
		(↑ SS LOC)=↓
		(↓ IND ID)=IND-loc
		(↓ COND REL)=OVERLAP
		(↓ COND ARG'.1)=(↓ IND)
		(↓ COND ARG'.2)=LOC-D
		(↑ SS POL)=1
-V3SGNEG	AFF	*
		(↑ FSTRUC TENSE)=PRESENT
		(↑ FSTRUC SUBJ FSTRUC NUM)=SG
		(↑ FSTRUC SUBJ FSTRUC PERSON)=3
		(↑ SS LOC)=↓
		(↓ IND ID)=IND-loc
		(↓ COND REL)=OVERLAP
		(↓ COND ARG'.1)=(↓ IND)
		(↓ COND ARG'.2)=LOC-D
		(↑ SS POL)=0
-VED	AFF	*
		$\left\{\begin{array}{l}(\uparrow \text{ FSTRUC TENSE})=\text{PAST}\\(\uparrow \text{ FSTRUC PARTICIPLE})=\text{PAST}\end{array}\right\}$
		(↑ SS LOC)=↓
		(↓ IND ID)=IND-loc
		(↓ COND REL)=PRECEDE
		(↓ COND ARG'.1)=(↓ IND)
		(↓ COND ARG'.2)=LOC-D
		(↑ SS POL)=1
-VINF	AFF	*
		$\left\{\begin{array}{l}(\uparrow \text{ FSTRUC TENSE})=\text{PRESENT}\\(\uparrow \text{ FSTRUC INF})=-\\\quad(\uparrow \text{ FSTRUC SUBJ NUM})=\text{SG}\\\quad[(\uparrow \text{ FSTRUC SUBJ PERSON })=3]\\(\uparrow \text{ FSTRUC INF})=+\end{array}\right\}$
A	DET	@A

		ROOT (↑ FSTRUC SPEC)=A (↑ FSTRUC NUM)=SG (↑ SS IND ID)=IND (↑ SS COND)=↓ (↑ SS SPEC)=A (↑ SS COND POL)=1 (↓ COND ARG'.1)=(↑ SS IND)
BARK	V	S-ED (↑ SS REL)=bark (↑ FSTRUC PRED)='BARK⟨(↑ FSTRUC SUBJ)⟩' (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS)
BITE	V	@(BITE- -VINF)
BITE-	V	ROOT { (↑ FSTRUC PRED)='BITE⟨(↑ FSTRUC SUBJ)⟩' (↑ FSTRUC PRED)='BITE⟨(↑ FSTRUC SUBJ)(↑ FSTRUC OBJ)⟩' (↑ SS ARG.2)=(↑ FSTRUC OBJ) } (↑ SS REL)=bite (↑ SS ARG.1)=(↑ FSTRUC SUBJ)
BITES	V	@(BITE- -V3SG)
BOY	N	S (↑ FSTRUC PRED)='BOY' (↑ SS COND REL)=boy
CANDIDATE	N	S (↑ FSTRUC PRED)='CANDIDATE' (↑ SS COND REL)=candidate
DO	AUX	@(DO- -VINF)
DO-	AUX	ROOT (↑ FSTRUC PRED)='DO⟨(↑ FSTRUC VCOMP)⟩(↑ FSTRUC SUBJ)' (↑ FSTRUC SUBJ)=(↑ FSTRUC VCOMP FSTRUC SUBJ) (↑ FSTRUC VCOMP FSTRUC CTYPE) (↑ FSTRUC VCOMP FSTRUC AUX) (↑ FSTRUC AUX)=+ (↑ SS REL)=(↑ FSTRUC VCOMP SS REL) (↑ SS ARG.1)=(↑ FSTRUC VCOMP FSTRUC SUBJ SS)

DOES	AUX	@(DO- -V3SG)
DOESN'T	AUX	@(DO- -V3SGNEG) (↑ FSTRUC NEG)=N'T
DOG	N	S (↑ FSTRUC PRED)='DOG' (↑ SS COND REL)=dog
DON'T	V	@DO (↑ NEG)=N'T
EVERY	DET	* (↑ FSTRUC SPEC)='EVERY' (↑ FSTRUC NUM)=SG (↑ SS IND ID)=IND (↑ SS)=↓ (↑ SS SPEC)=every (↑ SS COND POL)=1 (↓ COND ARG'.1)=(↑ SS IND)
HAS	V	@(HAVE- -V3SG)
HE-	N	ROOT (↑ FSTRUC PRED)='HE' (↑ FSTRUC NUM)=SG (↑ FSTRUC PERSON)=3 (↑ SS IND ID)=IND-e
HIM	N	@HE- (↑ FSTRUC CASE)=ACC
HUNT	V	S-ED (↑ SS REL)=HUNT (↑ FSTRUC PRED)='HUNT⟨(↑ FSTRUC SUBJ)(↑ FSTRUC OBJ) ⟩' (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS) (↑ SS ARG.2)=(↑ FSTRUC OBJ SS)
HURT	V	@(HURT- -VED)
HURT-	V	ROOT

		(↑ SS REL)=hurt (↑ FSTRUC PRED)='HURT⟨(↑ FSTRUC SUBJ)(↑ FSTRUC OBJ) ⟩' (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS) (↑ SS ARG.2)=(↑ FSTRUC OBJ SS)
IT	N	@IT-
IT-	N	ROOT (↑ FSTRUC PRED)='IT' (↑ FSTRUC NUM)=SG (↑ FSTRUC PERSON)=3 (↑ SS IND ID)=IND-e
JOHN	NPROP	* (↑ FSTRUC PRED)='JOHN' (↑ FSTRUC NUM)=SG (↑ FSTRUC PERSON)=3 (↑ SS IND-e)=JOHN
KICK	V	S-ED (↑ SS REL)=KICK (↑ FSTRUC PRED)='KICK⟨(↑ FSTRUC SUBJ)(↑ FSTRUC OBJ) ⟩' (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS) (↑ SS ARG.2)=(↑ FSTRUC OBJ SS)
MAN	N	* (↑ FSTRUC PRED)='MAN' (↑ SS COND REL)=man
NOT	NOT	*
OWN	V	S-ED (↑ FSTRUC PRED)='OWN⟨(↑ FSTRUC SUBJ)(↑ FSTRUC OBJ)⟩ ' (↑ SS REL)=OWN (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS) (↑ SS ARG.2)=(↑ FSTRUC OBJ SS)
PLUTO	NPROP	* (↑ FSTRUC PRED)='PLUTO' (↑ FSTRUC NUM)=SG (↑ FSTRUC PERSON)=3 (↑ SS IND-e)=Pluto

POLICEMAN	N	* (↑ FSTRUC PRED)='MAN' (↑ SS COND REL)=man
SHOOT-	V	ROOT (↑ FSTRUC PRED)='SHOOT⟨(↑ FSTRUC SUBJ)(↑ FSTRUC OBJ) ⟩' (↑ SS REL)=SHOOT (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS) (↑ SS ARG.2)=(↑ FSTRUC OBJ SS)
SHOOTS	V	@(SHOOT- -V3SG)
SHOT	V	@(SHOOT- -VED)
THE	DET	* (↑ FSTRUC SPEC)=THE (↑ FSTRUC DEF)=+ (↑ SS IND ID)=IND (↑ SS COND)=↓ (↑ SS SPEC)=THE (↑ SS COND POL)=1 (↓ ARG'.1)=(↑ SS IND)
WALK	V	S-ED (↑ FSTRUC PRED)='WALK⟨(↑ FSTRUC SUBJ)⟩' (↑ SS REL)=walk (↑ SS ARG.1)=(↑ FSTRUC SUBJ SS)
WHO	N	* (↑ FSTRUC PRED)='WHO' (↑ FSTRUC WH)=+

Remark. The issue of how polarity should be computationally determined is complicated. If there is no tense and no sentence negation, then the polarity should be 1, and it could be induced by the main verb. If there is negation, then it seems appropriate to say that the polarity is a function of the sign of the negation and the sign or polarity of the constituent which the negation applies to (*It is not not the case that Bill is tall*). It may be wrong to say that it is the tense that determines the polarity, because that would suggest that there is no polarity in tense-less clauses, such as infinitivals. The determination of the aspects of the *LOC*ation induced by tense could either be driven by the morpheme expressing the tense (as we do here), or by the occurrence of a particular value for the tense attribute in the f-structure. The latter might seem desirable since the tense may be morphologically complex and syntactically and morphologically discontinuous (cf. *I have not eaten*). The f-structure would then pick out the correct tense form for us. We have not done the work required to make a principled choice between these options.

This concludes the survey of how the morphological, phonological, syntactic and functional descriptions of linguistic form constrain the situation schemata for utterances. We now proceed to explain in detail how interpretations can be derived from the situation schemata.

CHAPTER III

INTERPRETING SITUATION SCHEMATA

In Chapter II we have described a map from *linguistic form* to a set of entities called *situation schemata.* The map is directed from linguistic form to situation schemata, hence the primacy of the former over the latter. As explained in I.1 of the introduction, the situation schema is a *theoretical notation*, a convenient way of summing up information from the linguistic form relevant for the semantic interpretation.

In this chapter we work out an interpretation in a system of situation semantics. We assume that the reader has some familiarity with Barwise and Perry's book, *Situations and Attitudes* (Barwise and Perry 1983). The presentation will, however, be reasonably self-contained. In Chapter IV we present a closer analysis of the mathematical structure of situation semantics.

In situation semantics, the *meaning* of a (simple declarative) sentence φ is a *relation* between an *utterance situation* u and a *described situation* s; we shall use *SIT.*φ, the situation schema associated with φ as given in Chapter II, to spell out the connection between u and s and write the basic meaning relation in the following form:

$$u[\![SIT.\varphi]\!]s$$

Notice that we have abbreviated SITSCHEMA.φ to *SIT.*φ.

The meaning relation decomposes into two parts:

- u is an *appropriate utterance situation* with respect to the constraints induced by *SIT.*φ, which we occasionally will write in the symbolism of *Situations and Attitudes* as

$$u \in MF_{[SIT.\varphi]}$$

– s is a *meaningful option* relative to the constraints induced by *SIT*.φ, in symbols

$$u\ MO_{[SIT.\varphi]}s$$

Note that there is a primary direction of flow of information. Given $u \in MF_{[SIT.\varphi]}$ the arrow points from u to s:

$$u \longrightarrow [\![SIT.\varphi]\!] \longrightarrow s$$

Note also the possibility of "inverse information", from the fact that s is a described situation relative to *SIT*.φ and some discourse situation u. We may obtain more specific information about u, see *Situations and Attitudes*. For our present purposes we shall concentrate on the "direct flow" and spell out how a given u and a given *SIT*.φ constrain or (partially) determine a described situation s.

In our subsequent discussion, the utterance situation will enter in a somewhat rudimentary and stereotyped way. We shall assume that the utterance situation decomposes into two parts:

d - discourse situation
c - the speaker's connection.

We will use d, c exactly as in *Situation and Attitudes*, and refer the reader to the discussion there on pages 120-127.

We assume that the discourse situation d uniquely determines (i.e. uniquely fills the roles of) the *speaker*, the *addressee*, the *expression uttered*, and the *location of the utterance*. We also assume that the discourse situation d provides us with sublocations to identify different occurrences of identical subexpressions of the expression uttered. We take the speaker's connection to be a uniquely determined (partial) map from appropriate lexical and morphological subparts of the expression uttered to the domain of primitives, i.e. the c of the utterance situation uniquely determines the referents of the referring subparts of the expression uttered as far as the utterance situation requires the referring subparts to be referring (the map c can be partial). Thus if u requires the subexpression α to have a referent in a described situation, then $c(\alpha)$ must be either a relation, an individual or a location in that situation, or $c(\alpha)$ could be another situation which could be used to determine how a role is to be filled in the situation s. We will return to this last point in our discussion of *loading* below. (See example (72) below.)

So much for the general format of the meaning relation $u[\![SIT.\varphi]\!]s$. We emphasize that our current specification of the utterance situation is rather minimal or sparse, more could and should be "fed into" the left side of the meaning relation to obtain, say, a better analysis of the flow of information in a discourse.

As the theory currently stands $SIT.\varphi$ contains mostly information directly computable from the syntax of the utterance φ. Thus, in analogy to the usual model theoretic approach, one might get the impression that $SIT.\varphi$ is some kind of "formula" derived from φ which is to be interpreted in the "model" given by the described situation s. But note that $SIT.\varphi$ also contains information in the LOC-part which is *not* part of the described situation, but which is an important link between the utterance situation u and the described situation s. (On this point see also Appendix A where the LOC-part gets an enlarged rôle.) The information computed in $SIT.\varphi$ is also not sufficient in every case to determine the relation $u[\![SIT.\varphi]\!]s$, we may have to encode further information to determine quantifier scope and readings of definite noun phrases; see the discussion of *Q.mode* in section III.2, and the discussion of *RMODE* in section III.1. Thus in a more definite version of the theory we foresee an expanded format for *situation schemata*, more symmetric in the information that it presents about u and s, thus emphasizing the *relational theory of meaning* underlying our work. However, what we can compute at the moment is $SIT.\varphi$; thus our immediate concern is to spell out in detail how $SIT.\varphi$ mediates between u and s.

1 THE ART OF INTERPRETATION

We will next give a series of examples which will teach the reader the *art* of unraveling the meaning relation $d, c[\![SIT.\varphi]\!]s$ for sentences φ of the grammar presented in Chapter II. Then we will give a more precise inductive definition of the relation. And we will conclude by discussing a "donkey-sentence", which we take as breaking the format of the standard (unary) generalized quantifiers. But this is going one step ahead of our account.

An "atomic" sentence φ, i.e. a sentence expressing a basic (located or unlocated) fact, such as

(57) John kicked Pluto

(58) Pluto was kicked by John

has a situation schema of the form

$$(59) \qquad SIT.\varphi \quad \begin{bmatrix} REL & r \\ ARG.1 & a \\ ARG.2 & b \\ LOC & - \\ POL & i \end{bmatrix}$$

where r names or can be anchored to a relation, a and b to individuals, and $i (\in \{0,1\})$ gives the polarity of the fact. LOC is a function which anchors the described fact relative to a discourse situation d, c. LOC will have the general format in (60) (see definition (17) in Chapter II).

$$(60) \qquad \begin{bmatrix} IND & IND.\alpha \\ COND & \begin{bmatrix} REL & r \\ ARG.1 & [\,]_\alpha \\ ARG.2 & \dot{l}_0 \end{bmatrix} \end{bmatrix}$$

where $IND.\alpha$ is an indeterminate for a location, r denotes one of the basic structural relations on the set Λ, and $\dot{l}_0$ is another location indeterminate. Tense can be interpreted in many ways; here we take a rather straightforward referential and "unlocated" point of view, and say that (the partial function) g *anchors the location of* $SIT.\varphi$, $SIT.\varphi.LOC$, in the discourse situation d,c if

$$g(\dot{l}_0) = l_d$$
$$c(r), g(IND.\alpha), l_d; 1$$

where l_d is the discourse location and $c(r)$ is the relation on Λ given by the speaker's connection c.

If the expression φ has a (morphological) subunit $\varphi\prime$, marking the tense of the expression, we may want to add the further constraint that $g(\mathit{IND}.\alpha) = c(\varphi')$.

Remark on notation. The correct form of (60) according to the rules of Chapter II is

$$(61)\qquad \left[\begin{array}{ll} \mathit{IND} & \mathit{IND}.\alpha \\ \mathit{COND} & \left[\begin{array}{ll} \mathit{REL} & r \\ \mathit{ARG.1} & \left[\begin{array}{ll} \mathit{IND} & [\;]_\alpha \end{array}\right] \\ \mathit{ARG.2} & \left[\begin{array}{ll} \mathit{IND} & i_0 \end{array}\right] \end{array}\right] \end{array}\right]$$

But we have and will adhere to an implicit bracketing convention which allows us to replace a box [*IND IND.α*] by *IND.α* when this box is the value of an *ARG.i*. In a similar way we write [*ARG.1 John*] rather than the "correct" [*ARG.1* [*IND John*]] in (62). There is no loss of information.

The situation schema corresponding to the sentences (60) and (61) is now:

$$(62)\qquad \mathit{SIT}.\varphi_1 \quad \left[\begin{array}{ll} \mathit{REL} & \mathit{kick} \\ \mathit{ARG.1} & \mathit{John} \\ \mathit{ARG.2} & \mathit{Pluto} \\ \mathit{LOC} & \left[\begin{array}{ll} \mathit{IND} & \mathit{IND.1} \\ \mathit{COND} & \left[\begin{array}{ll} \mathit{REL} & \prec \\ \mathit{ARG.1} & [\;]_1 \\ \mathit{ARG.2} & i_0 \end{array}\right] \end{array}\right] \\ \mathit{POL} & 1 \end{array}\right]$$

In this "atomic" situation

$$d, c[\![SIT.\varphi_1]\!]s$$

if and only if $d, c \in MF_{[SIT.\varphi_1]}$ (i.e. d, c is an appropriate discourse situation relative to $SIT.\varphi_1$) and there exists an anchor g on $SIT.\varphi_1.LOC$ such that

in s : at $g(\mathit{IND.1})$: *c(kick), c(John), c(Pluto);1*

The reader will notice how this clause parallels the standard interpretation of atomic formulas in predicate logic; see section IV.4 where we explain in detail how the present interpretation is related to the model-theoretic approach.

The reader should note that in these examples we have adopted a rather simple-minded treatment of names: The speaker's connection picks out a unique referent in the described situation. It is, of course, possible to work into our interpretation a more sensitive treatment along the lines suggested in *Situations and Attitudes* (pp. 165-168).

Our next example is the sentence

(63) John kicked a dog

A noun phrase such as *a* α (*an* α or *the* α) can be given either a *generalized quantifier* interpretations or a *singular NP-reading.* In either case we generate the following situation schema.

(64) $$\mathit{SIT}.\varphi_3 \begin{bmatrix} \mathit{REL} & \mathit{kick} \\ \mathit{ARG.1} & \mathit{John} \\ \mathit{ARG.2} & \begin{bmatrix} \mathit{IND} & \mathit{IND.1} \\ \mathit{SPEC} & \mathit{A} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \mathit{dog} \\ \mathit{ARG.1} & [\,]_1 \\ \mathit{POL} & 1 \end{bmatrix} \end{bmatrix} \\ \mathit{LOC} & \begin{bmatrix} \mathit{IND} & \mathit{IND.2} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \prec \\ \mathit{ARG.1} & [\,]_2 \\ \mathit{ARG.2} & i_0 \end{bmatrix} \end{bmatrix} \\ \mathit{POL} & 1 \end{bmatrix}$$

With the singular NP reading we get:

$$d, c[\![SIT.\varphi_3]\!]s$$

if and only if $d, c \in MF_{[\![SIT.\varphi_3]\!]}$ and there exists an anchor g on the $SIT.\varphi_3.LOC$ and an extension $g' \supseteq g$ which anchors $SIT.\varphi_3.ARG.2$ in s, i.e.

in s : *c(dog), g′(IND.1);1*

such that

in s : at *g(IND.2)* : *c(kick), c(John), g′(IND.1);1*

Notice that we have used an unlocated fact in the interpretation of *ARG.2* in (64). This corresponds to the lack of tense markers in the subpart “a dog” of the sentence (63).

The sentence

(65) Every boy kicked Pluto

has the following associated situation schema:

$$(66) \qquad SIT.\varphi_4 \begin{bmatrix} REL & kick \\ ARG.1 & \begin{bmatrix} IND & IND.1 \\ SPEC & EVERY \\ COND & \begin{bmatrix} REL & boy \\ ARG.1 & [\,]_1 \\ POL & 1 \end{bmatrix} \end{bmatrix} \\ ARG.2 & Pluto \\ LOC & \begin{bmatrix} IND & IND.2 \\ COND & \begin{bmatrix} REL & \prec \\ ARG.1 & [\,]_2 \\ ARG.2 & \dot{l}_0 \end{bmatrix} \end{bmatrix} \\ POL & 1 \end{bmatrix}$$

We may give a rather direct reading as follows:

$$d, c[\![SIT.\varphi_4]\!]s$$

if and only if $d, c \in MF_{[SIT.\varphi_4]}$ and there exists an anchor g on $SIT.\varphi_4.LOC$ relative to d, c such that for every extension $g' \supseteq g$ which anchors $SIT.\varphi_4.ARG.1$ in s, i.e. such that in s : c(boy), g′(*IND.1*);*1*, we have

in s : *at g(IND.2)* : *c(kick),g′(IND.1),c(Pluto);1*

Let us, however, be a bit more systematic at this point. The *ARG.1* in (66) is really a *notation* for a *restricted quantifier*,

$$\forall x(boy(x) \rightarrow \ldots)$$

or

$$\forall x \in boy(\ldots).$$

We see that *ARG.1.IND* introduces the quantified variable, *ARG.1.SPEC* the nature of the quantifier, and *ARG.1.COND* specifies the domain of variation of the quantifier. And this is what we have written out in the "direct reading" above.

For the more formal development in section III.2 we shall use the mechanism of *generalized quantifiers* (see Barwise and Cooper (1981) and Fenstad (1979)). As a preparation let us in this simple context see how this mechanism works.

From $SIT.\varphi_4$ we get two "fact schemata", one from the *REL* of $SIT.\varphi_4$

$$C_1 : \text{at} \quad \mathit{IND.2} : \mathit{kick}, \mathit{IND.1}, \mathit{Pluto}; \mathit{1}$$

and one from the *REL* of *ARG.1.COND*

$$C_2 : \mathit{boy}, IND.1; 1$$

From fact schemata we introduce "abstracts" of the form:

$$< \mathit{IND.1} | C_1 >$$
$$< \mathit{IND.1} | C_2 >$$

In more complicated situations a fact schema may contain more indeterminates, hence determine different abstracts. Each abstract determines a "parametric set". We assume a referential reading of tense, hence we assume that there is given an anchor g on $SIT.\varphi_4.LOC$ relative to a discourse situation d, c. Then for each abstract and each specification d, c, s we have the sets:

$$_{d,c,s}X_{C_1} = \{a | d, c[\![C_1]\!]s, a\}$$

and

$$_{d,c,s}X_{C_2} = \{a | d, c[\![C_2]\!]s, a\}$$

with the natural reading of the defining conditions:

$$d, c[\![C_1]\!]s, a \quad \text{iff} \quad \text{in } s : \text{at } \mathit{g(IND.2)} : \mathit{c(KICK),a,c(PLUTO); 1}$$

and

$$d, c[\![C_2]\!]s, a \quad \text{iff} \quad \text{in } s : \mathit{c(BOY),a; 1}$$

From the *SPEC* of *ARG.1* and the two abstracts we can form the "complex fact schema"

$$EVERY(<IND.1|C_2>)(<IND.1|C_1>)$$

which, since it has no further free indeterminates (*IND.1* being bound by the abstraction), has an interpretation in s relative to a discourse situation d, c as follows

$$d, c[\![\ EVERY(<IND.1|C_2>)(<IND.1|C_1>)]\!]s$$

if and only if

$$_{d,c,s}X_{C_2} \subseteq {}_{d,c,s}X_{C_1}$$

the inclusion relation between sets of individuals being determined by the *SPEC*, *EVERY* of *ARG.1*.

Thus we have the alternative but equivalent reading for sentence (66):

$$d, c[\![SIT.\varphi_4]\!]s$$

if and only if $d, c \in MF_{[\![SIT.\varphi_4]\!]}$ and there exists an anchor g on $SIT.\varphi_4.LOC$ such that relative to this anchor

$$d, c[\![\ EVERY(<IND.1|C_2>)(<IND.1|C_1>)]\!]s$$

But what to do with the sentence (67)

(67) Every boy kicked a dog

which could be scope ambiguous? The $SIT.\varphi_5$ does not carry enough information to choose between the possible readings, which, in our opinion, is as it should be. We need to add information, a scope specification, in order to complete the interpretation. We will return to this below.

Remark. We have in this work concentrated on generalized quantifier readings. However, for many purposes a singular NP reading may be more appropriate. This has been emphasized by J. Barwise in his work on situation semantics, see e.g. Barwise (1985). The present format can easily be extended to include singular NP readings, e.g. using the mechanism introduced by J.T. Lønning (1985) in his work on plurals and mass nouns.

We next discuss a simple example of a sentence with a restrictive relative clause. (As we are here illustrating a "point of principle" we close our eyes to the fact that the following sentence may have a preferred generic reading; we will insist on a singular NP-reading.)

(68) A dog who barks doesn't bite

This sentence has the following associated situation schema

(69)
$$
\mathit{SIT}.\varphi_6
\begin{bmatrix}
\mathit{REL} & \mathit{bite} \\
\mathit{ARG.1} & \begin{bmatrix} \mathit{IND} & \mathit{IND.1} \\ \mathit{SPEC} & A \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \mathit{dog} \\ \mathit{ARG.1} & [\,]_1 \\ \mathit{POL} & 1 \end{bmatrix} \\ \mathit{SITSCHEMA} & \begin{bmatrix} \mathit{REL} & \mathit{bark} \\ \mathit{ARG.1} & [\,]_1 \\ \mathit{LOC} & \begin{bmatrix} \mathit{IND} & \mathit{IND.2} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \circ \\ \mathit{ARG.1} & [\,]_2 \\ \mathit{ARG.2} & i_0 \end{bmatrix} \end{bmatrix} \\ \mathit{POL} & 1 \end{bmatrix} \end{bmatrix} \\
\mathit{LOC} & \begin{bmatrix} \mathit{IND} & \mathit{IND.3} \\ \mathit{COND} & \begin{bmatrix} \mathit{REL} & \circ \\ \mathit{ARG.1} & [\,]_3 \\ \mathit{ARG.2} & i_0 \end{bmatrix} \end{bmatrix} \\
\mathit{POL} & 0
\end{bmatrix}
$$

Recalling our global referential point of view of tense locations, we arrive at the following condition, using the singular NP-reading for

ARG.1.

$$d, c[\![SIT.\varphi_6]\!]s$$

if and only if $d, c \in MF_{[SIT.\varphi_6]}$ and there is an anchor g on $SIT.\varphi_6.LOC$ and $SIT.\varphi_6.ARG.1.SITSCHEMA.LOC$, i.e.

$$g(\dot{l}_0) = l_d,\ g(IND.2) \circ l_d,\ g(IND.3) \circ l_d$$

and an extension $g' \supseteq g$ anchoring $SIT.\varphi_6.ARG.1$ in s, i.e.

in s : *c(dog)*, $g'(IND.1)$;1
in s : at $g(IND.2)$: *c(bark)*, $g'(IND.1)$; 1

such that

in s : at $g(IND.3)$: *c(bite)*, $g'(IND.1)$; *0*

Notice that we have not required *g(IND.2)* and *g(IND.3)* to overlap. With present tense this could be reasonable, but not necessarily true. With past tense there is no assumption of co-referentiality of different occurrences of past tense markers in a sentence:

(70) The man who ran down University Avenue sweated profusely

(71) My friend who visited Italy also went to Spain

We next turn to an example with a definite description. Singular NP-reference is treated at great lengths in *Situations and Attitudes*; for illustrative purposes we restrict attention to referential and attributive uses in order to show how a *loading mechanism* can be adjoined to the situation schema. Our example is

(72) The boy kicked Pluto

To give the interpretation of this sentence it proves useful to enlarge the format of the general situation schema to include a slot for a situation indeterminate.

$$(73) \qquad SIT.\varphi \begin{bmatrix} SIT & - \\ REL & - \\ ARG.1 & - \\ \cdot & \cdot \\ \cdot & \cdot \\ ARG.n & - \\ LOC & - \\ POL & - \end{bmatrix}$$

With this goes a new kind of indeterminate, *SIT.1,...,SIT.n*. The same expansion could be made to the various *COND*'s and *SITSCHEMA*'s of the occurring *ARG*'s. The situation schema associated with (72) could be:

$$(74) \qquad SIT.\varphi_7 \begin{bmatrix} REL & kick \\ ARG.1 & \begin{bmatrix} IND & IND.1 \\ SPEC & THE \\ COND & \begin{bmatrix} SIT & SIT.1 \\ REL & boy \\ ARG.1 & [\ ]_1 \\ POL & 1 \end{bmatrix} \end{bmatrix} \\ ARG.2 & Pluto \\ LOC & \begin{bmatrix} IND & IND.2 \\ COND & \begin{bmatrix} REL & \prec \\ ARG.1 & [\ ]_2 \\ ARG.2 & \dot{l}_0 \end{bmatrix} \end{bmatrix} \\ POL & 1 \end{bmatrix}$$

We note that only the *COND* of *ARG.1* has been expanded. This suffices for our purposes, and it is straightforward to write down general rules supplementing our previous construction of situation schemata.

A *mode of reference*, an *RMODE*, is a specification which to every occurring *ARG.COND.SIT* associated to an *ARG.SPEC.THE*, gives a value 0 or 1. This would give $d, c[\![SIT.\varphi_7]\!]s$ relative to the *RMODE R* if and only if $d, c \in MF_{[\![SIT.\varphi_7]\!]}$ and there is an anchor g on the *LOC* of $SIT.\varphi_7$ and on the *SIT.1* of $SIT.\varphi_7$, i.e.

$$g(SIT.1) = s \text{ if } R(SIT.1) = 0$$

$$g(SIT.1) = c(\textit{the boy}) \text{ if } R(SIT.1) = 1$$

(where *c*(*the boy*) is the situation used by the speaker to evaluate the phrase *the boy*) and an extension $g' \supseteq g$ anchoring *ARG.1* in *g(SIT.1)*, i.e. *g(IND.1)* is the *unique* individual satisfying

in *g(SIT.1)* : *c(boy)*, *g′(IND.1);1*

such that

in s : at *g(IND.2)* : *c(kick)*, *g′(IND.1)*, *c(Pluto);1*

Note that the *RMODE* has not been made part of the situation schema but is an added piece of information derived from a possibly larger context than the uttered expression itself, see our concluding remarks in the introduction to this section. We shall return to this point in III.3.

We turn to the final example in this section.

(75) The boy who kicked Pluto hurt him

To simplify a bit we omit the *RMODE* mechanism and propose the following situation schema:

(76) $SIT.\varphi_8$

$$\left[\begin{array}{ll}
\mathit{REL} & \mathit{hurt} \\
\mathit{ARG.1} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.1} \\
\mathit{SPEC} & \mathit{THE} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{boy} \\
\mathit{ARG.1} & [\,]_1 \\
\mathit{POL} & 1
\end{array}\right] \\
\mathit{SITSCHEMA} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{kick} \\
\mathit{ARG.1} & [\,]_1 \\
\mathit{ARG.2} & \mathit{Pluto} \\
\mathit{LOC} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.2} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \prec \\
\mathit{ARG.1} & [\,]_2 \\
\mathit{ARG.2} & i_0
\end{array}\right]
\end{array}\right] \\
\mathit{POL} & 1
\end{array}\right]
\end{array}\right] \\
\mathit{ARG.2} & \mathit{IND.3} \\
\mathit{LOC} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.4} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \prec \\
\mathit{ARG.1} & [\,]_4 \\
\mathit{ARG.2} & i_0
\end{array}\right]
\end{array}\right] \\
\mathit{POL} & 1
\end{array}\right]$$

This schema has a free indeterminate, the *IND.3* of $SIT.\varphi_7.ARG.2$. Thus we will have to specify the meaning relation $d, c[\![SIT.\varphi_8]\!]s$ relative to an anchor f on the free indeterminates of $SIT.\varphi_8$. We then have

$$d, c[\![SIT.\varphi_8]\!]s$$

relative to the anchor f if and only if $d, c \in MF_{[SIT.\varphi_8]}$ and there is an anchor g on the *LOC*'s of *SIT*.φ_8 and an extension $g' \supseteq g$ anchoring *SIT*.φ_8.*ARG.1* in s, i.e. g'(*IND.1*) is the *unique* individual satisfying

in s : *c(boy)*, g'*(IND.1);1*
in s : at *g(IND.2)* : *c(kick)*, g'*(IND.2)*, *c(Pluto); 1*

such that

in s : at *g(IND.4)* : *c(hurt)*, g'*(IND.1)*, *f(IND.3);1*

Thus we have a full specification of the meaning relation. But how do we connect "him" and "Pluto"? If it so happens that

(i) $f(IND.3) = c(him) = c(Pluto)$

then "him" refers to "Pluto" and this is correctly recorded by the anchor *f*. If on the other hand

(ii) $f(IND.3) = c(him) \neq c(Pluto)$

then "him" has an external reference, and, once more, this is caught by the anchor *f*.

But this cannot be counted as a satisfactory theory of anaphoric linking, because accidental coreference is not the same as the intended identity. It would be better to effect the binding by adjoining a further constraint equation setting *IND.2* equal to *IND.3*. Then the schema would have no free indeterminate and "him" is forced to refer to "Pluto", regardless of what external facts say. If *IND.3* is kept free, we could have either (i) or (ii) above.

It is not, however, obvious that we on syntactic grounds alone are justified in adding further constraints. If we had the sentence

(77) The boy who kicked Pluto hurt himself

"himself" will refer to "the boy". In this case we assume that the basic *SITSCHEMA* would be enriched by a further constraining equation setting the indeterminate of *ARG.1* equal to the indeterminate of *ARG.2.*

The general case will need more careful study. It is not our purpose here to present a theory of anaphoric reference. We would only like to show that the format of situation schemata is flexible enough to accommodate various solutions whether obtained by co-indexing, i.e. using the same indeterminates in different positions, or obtained by an appeal to the mechanism of speaker's connection, i. e. to information located in the utterance situation. We note that a more elaborate treatment of names would be advantageous for a co-indexing mechanism, introducing indeterminates also in connection with names to facilitate the cross-linkings. The reader should consult Gawron and Peters (to appear) for further discussion of these matters.

2 THE INDUCTIVE DEFINITION OF THE MEANING RELATION

With these examples we trust that the reader has been sufficiently instructed in the *art* of interpretation. It will now be our duty to turn to the inductive definitions of the meaning relations $d, c[\![SIT.\varphi]\!]s$ relative to some scope specification.

We shall use the following sentence to illustrate the inductive definition

(78) Every policeman hunts a man who shot a candidate

The associated situation schema is

(79) $SIT.\varphi_9$

$$
\left[\begin{array}{ll}
\mathit{REL} & \mathit{hunt} \\
\mathit{ARG.1} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.1} \\
\mathit{SPEC} & \mathit{EVERY} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{policeman} \\
\mathit{ARG.1} & [\,]_1 \\
\mathit{POL} & 1
\end{array}\right]
\end{array}\right] \\
\mathit{ARG.2} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.2} \\
\mathit{SPEC} & \mathit{A} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{man} \\
\mathit{ARG.1} & [\,]_2 \\
\mathit{POL} & 1
\end{array}\right] \\
\mathit{SITSCHEMA} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{shoot} \\
\mathit{ARG.1} & \mathit{IND.2} \\
\mathit{ARG.2} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.3} \\
\mathit{SPEC} & \mathit{A} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{candidate} \\
\mathit{ARG.1} & [\,]_3 \\
\mathit{POL} & 1
\end{array}\right]
\end{array}\right] \\
\mathit{LOC} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.4} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \prec \\
\mathit{ARG.1} & [\,]_4 \\
\mathit{ARG.2} & l_0
\end{array}\right]
\end{array}\right] \\
\mathit{POL} & 1
\end{array}\right]
\end{array}\right] \\
\mathit{LOC} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.5} \\
\mathit{COND} & \left[\begin{array}{ll}
\mathit{REL} & \circ \\
\mathit{ARG.1} & [\,]_5 \\
\mathit{ARG.2} & l_0
\end{array}\right]
\end{array}\right]
\end{array}\right]
$$

There are at least three readings of (78) depending upon how quantifier scope is specified. There is one reading which follows the linear order of the words in the sentence: "For every policeman there is a man who shot a candidate and the policeman hunts this man." A second reading gives "a man who shot a candidate" wide scope: "There is a man who shot a candidate and every policeman hunts him." A third reading gives "a candidate" wide scope: "There is a candidate such that every policeman hunts a man who shot him."

We leave it to the reader to imagine contexts in which these readings are plausible. Our task here is to spell out an inductive definition of $d, c[\![SIT.\varphi]\!]s$ which follows the format of definition (27) in II.4 and which makes these readings possible. In doing this we elaborate on the machinery introduced in connection with sentence (74) to get the generalized quantifier reading of *every*. To simplify, we assume in the following discussion that there is given a global anchor g on the *LOC*'s of the *SIT.φ*.

1. A given *SIT.φ* generates various atomic or simple "fact schemata"; there is a "global" one coming from the *SIT.φ* itself, and there are others introduced by the various *COND*'s and *SITSCHEMA*'s associated with the occurring *ARG*'s. We form these fact schemata using the indeterminates introduced with the *ARG*'s of the *SIT.φ*, the *COND*'s and the *SITSCHEMA*'s.

Example. From $SIT.\varphi_9$ we get the fact schemata:

C_1 : at *IND.5* : *hunt,IND.1,IND.2;1*
C_2 : *policeman, IND.1; 1*
C_3 : *man, IND.2; 1*
C_4 : at *IND.4* : *shoot, IND.2, IND.3; 1*
C_5 : *candidate, IND.3; 1*

In our example the fact schemata have only indeterminates at the argument slots. In analogy with atomic formulas of predicate logic we could also fill the argument slots with names. The sentence

John hunts a man who shot a candidate

would generate the fact schemata

C'_1 : at *IND.5* : *hunt,John,IND.2;1*
C'_3 : *man, IND.2; 1*
C'_4 : at *IND.4* : *shoot, IND.2, IND.3; 1*
C'_5 : *candidate, IND.3; 1.*

2. From fact schemata we can form "abstracts" of the form

$$<IND.i,\ldots,IND.j \mid C>$$

where C is one of the fact schemata or the conjunction of the fact schemata of a *COND* and an associated *SITSCHEMA*. The variables *IND.i,...,IND.j* become ***bound*** in the abstract; C may have some remaining *free* indeterminates.

An abstract determines a "parametric set" in the following way: Let d, c be a discourse situation and s a described situation. If f is an anchor on the remaining free indeterminates of C, we let

$${}_{d,c,s}X^f_C = \{< a_i, \ldots, a_j > \mid d, c[\![C]\!]s, f, a_i \ldots, a_j\}$$

with the "obvious" interpretation of $d, c[\![C]\!]s, f, a_i \ldots, a_j$ (see the comment following example (66) above).

The machinery introduced in 1 and 2 is sufficient to account for the generalized quantifier reading of simple sentences such as ***(Every boy kicked Pluto)***, see example (65). But quantifiers may occur within the scope of other quantifiers, and we are led from the atomic or simple fact schemata of 1 to a more general notion of fact schema. The inductive definition is as follows:

(i) Atomic schemata are ***fact schemata.***

(ii) Let Q be a specifier (i.e. quantifier), *IND.i* an indeterminate, and C',C'' ***fact schemata***, then

$$Q(< IND.i \mid C' >)(< IND.i \mid C'' >)$$

is a ***fact schema.***

(iii) If C is an ***atomic fact schema*** derived from *COND*, and C' is a ***fact schema***, then C ***and*** C' is a ***fact schema.***

Notice that we have extended the notion of "abstract" $<$*IND.i,...,IND.j*$\mid C >$ from atomic to general fact schemata.

Example. Using (iii) we can form the schema C_3 *and* C_4, and then the abstract $< IND.2 \mid C_3 \text{ and } C_4 >$. using (ii) we then get

$$C_1' = A(< IND.2 \mid C_3 \text{ and } C_4 >)(< IND.2 \mid C_1 >).$$

We assume that the reader is somewhat familiar with the theory of generalized quantifiers and thus would expect the following interpretation of C_1' relative to a discourse situation d, c, a described situation s, and an anchor f on the remaining free indeterminates, *IND.3* and *IND.1*, of C_1':

$$d, c[\![C_1']\!]s, f \quad \text{iff} \quad {}_{d,c,s}X^f_{C_3 \text{ and } C_4} \cap {}_{d,c,s}X^f_{C_1} \neq \emptyset,$$

where

$$a \in {}_{d,c,s}X^f_{C_3 \text{ and } C_4}$$

iff

in s : at g(IND.4) : c(shoot), a, f(IND.3); 1

and

in s : c(man), a; 1;

and

$$a \in {}_{d,c,s}X^f_{C_1}$$

iff

in s : at g(IND.5) : c(hunt), f(IND.1),a;1.

But we owe our reader a precise inductive definition. Let d, c and s be given:

(i) Let $C =$ at *IND.i* : *r,. . ., IND.j,. . .,a,. . .;pol* be an atomic fact schema and f an anchor on the free indeterminates of C, then

$$d, c[\![C]\!]s, f$$

iff

in s : at g(IND.i) : c(r),. . .,f(IND.j),. . .,c(a),. . .;pol.

(Remember that we have assumed that a global anchor g on the *LOC*'s of the *SITSCHEMA* is given.)

(ii) Let $C = Q(< IND.i \mid C' >)(< IND.i \mid C'' >)$ and let f be an anchor of the free indeterminates of C, then

$$d, c[\![C]\!]s, f$$

iff

$$[\![Q]\!](\{a \mid d, c[\![C']\!]s, f \cup \{< IND.i, a >\}\},$$
$$\{a \mid d, c[\![C'']\!]s, f \cup \{IND.i, a >\}\}),$$

where $[\![Q]\!]$ is the set-operation interpreting the quantifier (e.g. $[\![EVERY]\!]$ is $\subseteq$ and $[\![A]\!]$ is $\cap$), and $f \cup \{< IND.i, a >\}$ is an anchor $f' \supseteq f$ such that $f'(IND.i) = a$.

(iii) Let $C = C'$ *and* C'' and let f be an anchor on the free indeterminates of C, then

$$d, c[\![C]\!]s, f \quad \text{iff} \quad d, c[\![C']\!]s, f \quad \text{and} \quad d, c[\![C'']\!]s, f.$$

(Note that in connection with the last clause we should have used the more precise expression: Let f be an anchor on a set of indeterminates which includes the free indeterminates of C. C' and C'' need not include the same set of free indeterminates.)

Example. If we let ${}_{d,c,s}X_C^f$ denote the set

$$\{a \mid d, c[\![C]\!]s, \ f \cup \{< IND.i, a >\}\},$$

then the reader may use the formal definitions to verify that

$$d, c[\![A(< IND.2 \mid C_3 \ \ and \ \ C_4 >)(< IND.2 \mid C_1 >)]\!]s, f$$

indeed asserts that

$${}_{d,c,s}X_{C_3}^f \cap {}_{d,c,s}X_{C_4}^f \cap {}_{d,c,s}X_{C_1}^f \neq \emptyset,$$

where f is an anchor on *IND.1* and *IND.3*.

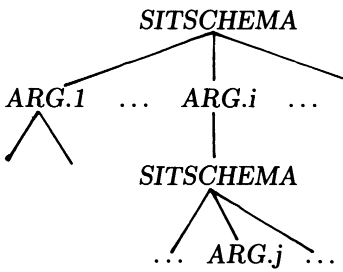
Remark. Our use of abstracts, $< IND.i, \ldots, IND.j \mid C >$, and of parametric sets, ${}_{d,c,s}X_C^f$, corresponds closely to ordinary set abstraction or λ-abstraction. Since we are in a simple extensional context, there are no mathematical subtleties involved. Note also that our use of abstracts, $< IND.i \mid C >$, allows us to introduce what in situation semantics are called complex indeterminates or rôles.

The sentence (79) has an interpretation only if a scope specification is given. In analogy with the device of *RMODE*, we introduce an extra feature called *QMODE*, not as part of the situation schema, but as a piece of added information, which could come from the linear order of words in the sentence, stress, intonation, or from the larger discourse situation. We do not here argue for a particular theory of scope phenomena. We only want to exhibit a device which we would like to believe is flexible enough to accommodate various theories.

Remark. Precursors of the notion of *QMODE* can be found in Halvorsen (1983) and Cooper (1983). It also seems to be closely related to a mechanism introduced by G. Chierchia to treat quantifier scope within the context of categorial grammar, see Chierchia (to appear). In revising the original version of this manuscript we have also benefited from discussions with S. Peters and M. Gawron on problems connected with quantifier scope; they have adopted an approach closely related to ours, see Gawron and Peters (to appear).

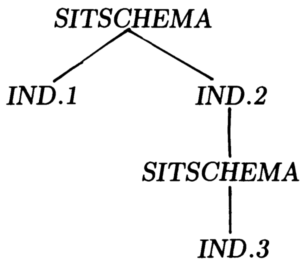
At the level of a $SIT.\varphi$, quantifiers can be introduced with the *ARG*'s of the *REL* of the $SIT.\varphi$, or they could be passed up through the *SIT-SCHEMA* of an *ARG*. Thus a *QMODE* is a specification which for each $SIT.\varphi$ and for the various *SITSCHEMAs*, indicates which indeterminates are to be quantified and in which order.

This gives the following general picture



Remember that the value of an *ARG.i* can either be a name, an indeterminate, or an embedded structure introducing a quantifier. In the latter case, a new *SITSCHEMA* may be introduced. Expanding on this (following the inductive clauses of definition (27) in II.4)) we get a finite tree structure as pictured above.

For the scope specification, the *QMODE*, we are only interested in the variables introduced in connection with a quantifier. In our example (79) we get the following simplified tree structure:



The *QMODE* will now have to specify in which order to discharge the indeterminates. We promised three readings in connection with (78):

QMODE.1 The quantifier associated with *IND.1* gets wide scope, and the quantifier associated with *IND.3* is "discharged" inside the scope of *IND.2*. This can be displayed as follows:

$$C_1 : EVERY(< IND.1|C_2 >); A(< IND.2|C_3 >).$$

$$C_4 : A(< IND.3|C_5 >).$$

QMODE.2 This reading gives the *IND.2*-quantifier scope over *IND.1*, but still evaluates *IND.3* inside the scope of *IND.2*:

$$C_1 : A(< IND.2|C_3 >); EVERY(< IND.1|C_2 >).$$

$$C_4 : A(< IND.3|C_5 >).$$

QMODE.3 Our third reading gives the *IND.3*-quantifier wide scope, but keeps the linear order between *IND.1* and *IND.2*:

$$C_1 : A(< IND.3|C_5 >); EVERY(< IND.1|C_2 >);$$
$$A(< IND.2|C_3 >).$$

$$C_4 : \emptyset.$$

Corresponding to *QMODE.1* we inductively build up the complex fact schema in the following steps:

$$C_4' : A(< IND.3|C_5 >)(< IND.3|C_4 >)$$

$$C_1' : A(< IND.2|C_3 \text{ and } C_4' >)(< IND.2|C_1 >)$$

$$C_1'' : EVERY(< IND.1|C_2 >)(< IND.1|C_1' >)$$

We leave *QMODE.2* to the reader and exhibit the steps leading to the complex fact schema corresponding to *QMODE.3*:

$$C_1' : A(< IND.2|C_3 \text{ and } C_4 >)(< IND.2|C_1 >)$$

$$C_1'' : EVERY(< IND.1|C_2 >)(< IND.1|C_1')$$

$$C_1''' : A(< IND.3|C_5 >)(< IND.3|C_1'' >)$$

The reader should verify, using the inductive definitions above, that the three *QMODE*s give the three intended readings of the sentence (78).

3 A REMARK ON THE GENERAL FORMAT OF SITUATION SCHEMATA.

As remarked on several occasions above the current format of a situation schema reflects what is computable from the utterance φ. As we have seen in III.2, more information may be necessary in order to spell out the meaning relation

$$u[\![SIT.\varphi]\!]s.$$

In particular, we found it necessary to consider a *QMODE*, to give a definite scope reading, and an *RMODE*, to allow for "resource situations" in determining the domain of quantification and to evaluate definite descriptions. We also argued that questions of anaphoric reference needed further information from the utterance situation for their resolution.

It would be possible to expand the format of a situation schema to include this kind of information. This would give a schema more balanced between the utterance situation and the described situation.

$$\begin{bmatrix} \text{DESCRIBED SITUATION} & SIT.\varphi \\ \text{UTTERANCE SITUATION} & \begin{bmatrix} \quad QMODE \\ \quad RMODE \\ \ldots \\ \ldots \end{bmatrix} \end{bmatrix}$$

But it is not at all clear what factors are relevant for determining particular settings for RMODE and QMODE and how these can be algorithmically determined. Thus we have, at this point, restricted ourselves to what is determined from the form of the utterance. However, in doing this we have gone beyond the information usually presented in linguistic analyses, e.g. in the f-structure of a sentence φ. We have included more in the *LOC*-part than is usually given in the *TENSE* attribute in LFG. In the treatment of locative prepositional phrases, which is outlined in Appendix A, even more information is introduced in the *LOC*. Work in progress by H. Sem (in prep.) develops yet another aspect of situation schemata in the analysis of temporal adverbs and anaphora. This work relates the present approach with the *Discourse Representation Systems* (DRS) as developed by Hans Kamp (1979, 1984).

4 GENERALIZING GENERALIZED QUANTIFIERS

We conclude this section by discussing the following sentence.

(80) Every boy who owns a dog kicks it

This type of sentence has the following situation schema, where we have presupposed an anaphoric reading.

(81) $SIT.\varphi_{10}$

$$
\left[\begin{array}{ll}
\mathit{REL} & \mathit{kick} \\
\mathit{ARG.1} & \left[\begin{array}{ll}
\mathit{IND} & \mathit{IND.1} \\
\mathit{SPEC} & \mathit{EVERY} \\
\mathit{COND} & \left[\begin{array}{ll} \mathit{REL} & \mathit{boy} \\ \mathit{ARG.1} & [\,]_1 \\ \mathit{POL} & 1 \end{array}\right] \\
\mathit{SITSCHEMA} & \left[\begin{array}{ll}
\mathit{REL} & \mathit{own} \\
\mathit{ARG.1} & [\,]_1 \\
\mathit{ARG.2} & \left[\begin{array}{ll} \mathit{IND} & \mathit{IND.2} \\ \mathit{SPEC} & \mathit{A} \\ \mathit{COND} & \left[\begin{array}{ll} \mathit{REL} & \mathit{dog} \\ \mathit{ARG.1} & [\,]_2 \\ \mathit{POL} & 1 \end{array}\right] \end{array}\right] \\
\mathit{LOC} & \left[\begin{array}{ll} \mathit{IND} & \mathit{IND.3} \\ \mathit{COND} & \left[\begin{array}{ll} \mathit{REL} & \circ \\ \mathit{ARG.1} & [\,]_3 \\ \mathit{ARG.2} & i_0 \end{array}\right] \end{array}\right] \\
\mathit{POL} & 1
\end{array}\right]
\end{array}\right] \\
\mathit{ARG.2} & \mathit{IND.2} \\
\mathit{LOC} & \left[\begin{array}{ll} \mathit{IND} & \mathit{IND.5} \\ \mathit{COND} & \left[\begin{array}{ll} \mathit{REL} & \circ \\ \mathit{ARG.1} & [\,]_5 \\ \mathit{ARG.2} & i_0 \end{array}\right] \end{array}\right] \\
\mathit{POL} & 1
\end{array}\right]
$$

This sentence has traditionally caused problems for a compositional analysis: how can the pronoun “it” be made to refer back to *a dog*?

Using the inductive procedure above the only way to do this would be to give *a dog* wide scope reading. But that is not satisfactory, since it would give *a dog* scope over *every man*, and it is a rare situation where there is a "universal" dog every man owns and kicks! As part of his DRS theory, H. Kamp has proposed an ingenious solution (Kamp 1984).

Here we approach the problem from a somewhat different perspective: *Not every generalized quantifier is unary.* This is commonplace in mathematical contexts, and it has also been suggested in more general philosophical or linguistic contexts, see e.g. Kalish and Montague (1964), and Altham and Tennant (1975). But it is an insight that tends to be overlooked. Thus we propose that the noun phrase

(82) Every boy who owns a dog

can be taken to signal a *binary* or *pair quantifier.* And there is at least two different readings, the unrestricted

(83) For every pair, boy(x) and every dog(y), such that x owns y...

or the dependent

(84) For every pair, boy(x) there is an associated dog $\pi(x)$ such that x owns $\pi(x)$

The reading in (84) selects for each boy some dog (or some set of dogs if π is allowed to be multiple-valued) owned by that boy. This is the contrast to the reading (83) where we assert something of every dog owned by the boy. The usage in (84) is common in mathematical contexts and one has a sophisticated theory of *selection operators* π to cope with it.

We do not aspire to this level of sophistication here, neither do we aim at giving a general theory for n-ary generalized quantifiers. We shall restrict ourselves to a discussion of sentence (80) and work out the unrestricted reading using the mechanism developed above for unary generalized quantifiers. This is, incidentally, the same reading that H. Kamp has proposed but, we emphasize, it is not the only possible reading.

From $SIT.\varphi_{10}$ we get the fact schema

$$C_1 : \text{at } \mathit{IND.4} : \mathit{kick}, \mathit{IND.1}, \mathit{IND.2}; 1$$

and from this the abstract

$$<\mathit{IND.1}, \mathit{IND.2}|C_1>$$

which gives the parametric set of pairs

$$_{d,c,s}X_{C_1} = \{< a, b > |d, c[\![C_1]\!]s\}$$

(where we have assumed given an anchor on the LOC's)

From the analysis of $ARG.1$, which introduces the pair quantifier, we get the schemata

$$C_2 : \text{at } \mathit{IND.3} : \mathit{own}, \mathit{IND.1}, \mathit{IND.2}; 1$$
$$C_3 : \mathit{boy}, \mathit{IND.1}; 1$$
$$C_4 : \mathit{dog}, \mathit{IND.2}; 1$$

From this we can form the complex schema

$$C_5 : C_2 \text{ and } C_3 \text{ and } C_4$$

and the abstract

$$<\mathit{IND.1}, \mathit{IND.2}|C_5>$$

which gives the parametric set of pairs

$$_{d,c,s}X_{C_5} = \{< a, b > |d, c[\![C_5]\!]s\}$$

Since we have opted for the unrestricted reading we simply get:

$d, c[\![SIT.\varphi_{10}]\!]s$ iff there is an anchor g on the LOC's of $SIT.\varphi_{10}$

such that

$$_{d,c,s}X_{C_5} \subseteq {}_{d,c,s}X_{C_1}$$

There is a minor variation of the notation that we have used in this example and which we will briefly mention in the section on the logic of situation semantics. Instead of forming the complex C_5 we could have formed a new type of abstract:

$$< IND.1|C_3, IND.2|C_2 >$$

Here we would use the COND's to restrict the range of variables and the *SITSCHEMA* to impose a further condition, i.e. we have the reading

> For every pair of boys and dogs such that the boy owns the dog...

The reading of $<IND.1, IND.2|C_5>$ would, in contrast, rather be

> For every pair of individuals such that one is a boy and the other is a dog and the first owns the second...

This concludes our discussion on how to interpret situation schemata in a system of situations semantics. Even if lengthy, the discussions have been fragmentary. In particular, we should like to investigate whether the notion of situation schemata is sufficiently "open" to handle current theories of attitude contexts, but that would be a topic for its own monograph. We now turn to the logic of situations.

CHAPTER IV

A LOGICAL PERSPECTIVE

In this part of the monograph, we are going to use a hierarchy of logical formal languages to learn more about the preceding semantic enterprise. As was explained in the general introduction, our purpose is not only the traditional search for complete systems of inference, but also to investigate a variety of semantic notions appearing in situation semantics. More specifically, Chapter IV.1 contains a general discussion of our semantic format, with a preview of the later model theory. The language hierarchy is developed in Chapter IV.2, with matching enrichments of the model structures at each step–introducing a variety of logical notions, questions and results. The main technical contribution is presented in Chapter IV.3, consisting of completeness and persistence theorems for several languages in our hierarchy, up to an expressive two-sorted one. Finally, Section IV.4 explains how the meaning relation, $d, c[\![SIT.\varphi]\!]s$, of Chapter III is related to the model-theoretic approach of this chapter.

1 THE MECHANICS OF INTERPRETATION

The basic ingredients of semantic interpretation for natural language structures have been explained in Chapter II and III, where situation schemata were used as a vehicle for establishing meaning relations. Of course, many things play a role in meaningful use of language. For instance, viewing the matter from the vantage point of a syntactic construct, relatively stable semantic background conventions interpret part of its vocabulary (common nouns, verbs, determiners and the like), more local speaker-dependent connections may govern the interpretation of, say, proper names and pronouns, while indexical expressions are even tied up with such volatile features as time and place of the actual utterance. This is the "production side" of the coin; the other being the situation described—or perhaps better, the type of situation described.

All this is mere common sense, and presumably most major semantic theories would agree on this kind of open ended list. Where they differ is in the ways of "cutting up" the picture. For instance, in Montague grammar, the standard format of interpretation looks like this:

> "in model $\mathcal{M}$, given a meaning assignment I to the vocabulary, as well as some assignment A to the free variables, the denotation of expression ϵ at index i is...."

where the index i is a package including a world w, a point in time t, plus (if necessary) items from the utterance situation (time of utterance, speaker, etc.) or larger "context." Although this format has certainly been chosen for convenience, rather than philosophical principle, the index package suggests a unity which hides very different functions. A brief comparison with the situation semantic set-up will bring this out.

In the latter approach, our semantic ingredients are grouped as follows:

> "in model $\mathcal{M}$, ϵ induces the following relation between utterance situation u (including "discourse situation" d and "speaker's connection" c) and a described situation s (plus location l), relative to a suitable anchor f on the indeterminates occurring in ϵ...."

Surely, there are striking similarities between the two schemes, even though the division of labor is different. For instance, the job of I is done by c, that of A partly by c, partly by f, while the various components of the index i are now scattered among d, s, and l. Even so, the latter picture, with its suggested "flow of information" between u and s, certainly has it attractive and suggestive features.

Actually, in the formal semantics developed in the area that is sometimes called "philosophical logic," various formats have been around, many of them relational, on the pattern of Tarski's paradigmatic truth definition for predicate logic. Tarski's notion may be written in the following notation:

$$< M, I > \models \epsilon[A]$$

(i.e., "formula ϵ is true in model $\mathcal{M}$ under interpretation I and assignment A"). Gradually, additional ingredients have been added, as the need arose. For instance, in a tense logic handling temporal indexicals, an interplay would occur between a "point of valuation" t and a contextual "point of speech" t_0, as in the following pattern:

$$< M, I >, t_o \models \epsilon[t; A]$$

Our formal languages below will be treated in a manner consonant both with situation semantics and their traditional model theory—at least, as far as the truth-conditional format is concerned.

There is also a more "local" sense in which situation semantics employs a "relational" format for expressing meanings. The denotational conditions attached to a specific expression ϵ show a characteristic "flattening"—as contrasted with the higher operator types of, say, Montague Grammar. For instance, transitive verbs correspond to conditions in two (individual) parameters, determiners with conditions in two (role) parameters, etc., where there used to be function-argument stacks.

As far as the "combinatorics" of meaning is concerned, there may not be all that much difference between this approach and the familiar lambda-calculations of earlier theories. Indeed, despite appearances, virtually algorithmic translations run in both directions. Nevertheless, this relational flattening often has heuristic virtues. One example is its suggestive use in recent logical studies of determiners. But also, it liberates us from a tendency which has been rather prominent on earlier approaches, viz. to think that full function hierarchies are the only proper semantic modeling for these combinatorial operations. And that is certainly not the case, nor is it desirable. This theme will be developed further below. We also recall our earlier point that information from linguistic form to interpretation does not flow only through syntactic structure, and that a flatter relational form may be more appropriate or at least convenient in this broader setting.

Given this praise, it may come as some surprise that the truth definitions in our logical part will turn out to follow a more traditional road, decomposing formulas inductively along their operator structure. Would it not be possible, say, to present propositional logic, properly viewed, in a flat relational format? The answer is positive. For instance, formulas might be viewed as being conditions on *polarity* indeterminates, with connectives contributing, e.g., as follows:

$[\![NOT(p_1, p_2)]\!]$ iff p_1 is the truth table complement of p_2

$[\![AND(p_1, p_2, p_3)]\!]$ iff p_1 is the truth table intersection of p_2, p_3

Then, *NOT (B AND NOT C)* would get its proper meaning by means of suitable equations identifying polarity indeterminates along the way. The reason why we have not chosen this approach here is that it did not seem to present obvious advantages in our case over the more traditional treatment.

One and the same semantic format can serve widely different model theories. For instance, the (intensional) typed language of Montague semantics may be interpreted in standard higher-order models (the way Montague himself presented his system), but also in Henkin-type "general models," without any "combinatorial explosion" in higher types (compare Gallin 1975). On the general model view, the language is treated essentially as a *many-sorted* one, which may even have models which are countable throughout. Notice also that, on this view, all types may be thought of as denoting "objects" of a certain sort—be it ones within a web of combinatorial ties with other kinds of objects. But also, more "intensional" categorial models have been proposed for Montague's logical language.

Likewise, situation semantics has already been involved with several suitors, including different set theories and model constructions within these. Our approach here is a many-sorted one. We isolate some basic kinds of entities that should occur in our models, together with some structure within and between these. In addition to this choice of a "model type," we shall investigate certain reasonable conditions to be imposed upon these. As was explained in Chapter III, the eventual picture will be that of tuples.

$$S \text{ (situations)}, \Lambda \text{ (locations)}, D \text{ (individuals)}, R \text{ (relations)}$$

One difference between situation semantics and earlier theories which has so far not been made explicit in our general discussion, is the issue of *partial interpretation.* Linguistic expressions may fail to impose a meaning relation between an utterance situation and a situation described, if certain prerequisites are not satisfied. In particular, situations may be "partial," lacking the necessary fact-types to make sense of certain sentences—and then an answer as to the truth of the sentence is not *YES* or *NO*, but indeterminate.

This phenomenon, recognized in earlier semantic theories, but usually treated as a marginal effect, is accorded central status in situation semantics. Thus, we might enrich the above notation $[\![\epsilon]\!]$ for the meaning relation with (non-exhaustive) polarity indications $[\![\epsilon]\!]^+$, $[\![\epsilon]\!]^-$: which indicates that ϵ is true [resp. false] relative to a model. (Alternatively, using polarity indeterminates, the whole matter can be subsumed under the above general notational schema, emphasizing the "suitability" clauses on anchor and connections.)

Actually, the phenomenon of partiality has been studied for a long time in modern logic, in such diverse areas as many-valued logic, recursion theory and modal logic. Some of the relevant insights will be utilized below. Nevertheless, it should be emphasized that the matter is not one of simple cosmetic surgery on earlier approaches. For instance, there are well-known technical obstacles to finding smooth partial versions of higher-order type theories—which we evade through our many-sorted perspective here. Eventually, this way out may not be fully satisfactory—in which case we should like to admit "just a little" type structure. But such problems are beyond the horizon of the present context.

As for suitable fine structure within the above models $< S, \Lambda, \mathcal{D}, R >$, two strategies are possible. One is to proceed to a direct study of "mathematical ontology", considering various interesting relations and operations on and between the various domains. For instance, in a partial perspective, the relation $\sqsubseteq$ of *extension* among situations becomes important, defined by (or at least implying) inclusion of associated facts. Depending on one's view of situations, $\sqsubseteq$ will be subject to various conditions. E.g., when situations are all fragments of some single covering reality, "convergence" will be plausible: every two situations are included in a third. Likewise, the locations, being extended spatio-temporal items, carry a lot of mathematical structure—which has received extensive attention in recent philosophical and logical literature (witness Kamp 1979, van Benthem 1982). Next, the individual domain D has structure encoded through the predicates in R, via the *in*-relation. Whether D also carries more independent "basic relations" remains to be seen. Finally, the relation domain R may be thought of as a relational algebra with suitable operations of complement, product, projection, etc., as in recent philosophical "theories of properties".

Another strategy is to approach the model structure taking our cues from the languages to be interpreted, formulating conditions on the structure thus obtained only when these have discernible effects upon inference within the language. We shall pursue the latter, more "minimalist" course here.

There is no real conflict between the two strategies, of course. For instance, in the case of time a two-pronged approach has been advocated and elaborated in van Benthem (1984)—and one could have proceeded in a similar fashion here.

Either way, on the analysis given here, we are abstracting away from a unique Reality (emphasized in the most recent phase of situation semantics), to a certain *type* of model with certain constraints. The resulting multiplicity, admitting semantic models that are not "the real thing," raises issues hotly debated by contemporary philosophers of language. We shall leave it with confidence in those capable hands.

2 A HIERARCHY OF FORMAL LANGUAGES

In this section we introduce successive systems of propositional, predicate and tense logic. The first part on propositional logic will be rather extensive, as it develops some fundamental themes common to all systems.

2.1 Propositional logic

2.1.1 The core system. Consider a propositional language with propositional atoms $p, q, r, \ldots$ and connectives $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (inclusive disjunction). Its formulas φ may be interpreted as global statements about some situation (or situation - location pair). Thus, of the full relation format $d, c[\![\varphi]\!]s$, we only need the described situation s and the speaker's connection c, which is supposed to give us a coupling between propositional atoms p and suitable relations $c(p)$in R. Moreover, no generality is lost by disregarding the location parameter for the moment. Thus, of the model structure, only s and part of R are relevant.

2.1.2 Truth definition. Let $M = \langle s, R, in\rangle$ be a model structure, and c a connection. Relative to M and c the stipulation for the basic cases becomes:

$$[\![p]\!]_s^+ \text{ iff } in\ s : c(p); 1 \qquad [\![p]\!]_s^- \text{ iff } in\ s : c(p); 0$$

The connectives are explained as follows (cf. Kamp 1983, Langholm 1983):

$$(\neg) \quad \begin{array}{lcl} [\![\neg\varphi]\!]_s^+ & \text{iff} & [\![\varphi]\!]_s^- \\ [\![\neg\varphi]\!]_s^- & \text{iff} & [\![\varphi]\!]_s^+ \end{array}$$

$$
\begin{array}{llll}
(\wedge) & [\![\varphi \wedge \psi]\!]_s^+ & \text{iff} & [\![\varphi]\!]_s^+ \textit{ and } [\![\psi]\!]_s^+ \\
 & [\![\varphi \wedge \psi]\!]_s^- & \text{iff} & [\![\psi]\!]_s^- \textit{ or } [\![\psi]\!]_s^- \\
(\vee) & [\![\varphi \vee \psi]\!]_s^+ & \text{iff} & [\![\varphi]\!]_s^+ \textit{ or } [\![\psi]\!]_s^+ \\
 & [\![\varphi \vee \psi]\!]_s^- & \text{iff} & [\![\varphi]\!]_s^- \textit{ and } [\![\psi]\!]_s^-
\end{array}
$$

This dual presentation is becoming standard (cf. also Veltman 1981). In essence, it amounts to employing a many-valued logic. For instance, the above explication could also be stated using *3-valued* truth tables:

$$
\begin{array}{ccc}
 & & \neg \\
\varphi & 1 & 0 \\
 & 0 & 1 \\
 & u & u
\end{array}
$$

$$
\begin{array}{cc|ccc}
 & & & \psi & \\
 & \wedge & 1 & 0 & u \\
\hline
\varphi & 1 & 1 & 0 & u \\
 & 0 & 0 & 0 & 0 \\
 & u & u & 0 & u
\end{array}
$$

$$
\begin{array}{cc|ccc}
 & & & \psi & \\
 & \vee & 1 & 0 & u \\
\hline
\varphi & 1 & 1 & 1 & 1 \\
 & 0 & 1 & 0 & u \\
 & u & 1 & u & u
\end{array}
$$

where u stands for "undefined."

The above truth-tables are well-known in the many-valued literature. (A recent study with many useful references is Visser 1984.) The third truth value u reflects the case when the formula is "under-defined" in a situation. For reasons of technical elegance, the dual possibility ("over-defined") is often introduced as well, leading to a *4-valued* semantics. The latter tolerates contradictions in a sense congenial to dialectical logicians and to students of the paradoxes. (A situation semantics allowing possibly "inconsistent situations" was studied in Langholm 1983. It will be used as a technical tool in IV.3 below.)

We now turn to some interesting features of our non-bivalent perspective.

Persistence

One important feature of information is its persistence ("monotonicity"): Once obtained in a situation, it should remain valid in its extensions. Within the present language, this shows up in the following result:

Lemma 1. *For all formulas φ, if $[\![\varphi]\!]^+_{s_1}$ and $s_1 \sqsubseteq s_2$, then $[\![\varphi]\!]^+_{s_2}$; and likewise for the negative version.*

Thus, all formulas of our language are both "true" and "false persistent". This will change later on, when non-monotone connectives are considered.

Persistence has been studied extensively in 3-valued logic— one of the highlights being a "functional completeness" theorem characterizing precisely those connectives guaranteeing the above lemma (Blamey 1980).

Varieties of Semantic Consequence

A 3-valued scheme offers options for the relation of semantic consequence. In situation semantics, two such notions have received a certain emphasis. The first is

Strong Consequence. For all M and s: *if* $[\![\varphi_1]\!]^+_s$ *and...and* $[\![\varphi_n]\!]^+_s$, *then* $[\![\psi]\!]^+_s$
(We use the notation: $\varphi_1, \ldots, \varphi_n \models_+ \psi$ for Strong Consequence.) There is also a more "eventual" notion of consequence:

Involvement. For all M and s: *if* $[\![\varphi_1]\!]^+_s$ *and...and* $[\![\varphi_n]\!]^+_s$,*then for some* $s' \sqsupseteq s$, $[\![\psi]\!]^+_{s'}$
As an example of the difference, $\varphi = p \vee \neg p$ is not strongly valid; but it would be in the involvement sense. Unlike the first, the latter notion is sensitive to assumptions about the contents of the larger universe—a topic that we shall not take up here.

These two notions do not exhaust all possibilities, however. One may also employ negative polarities, as in

Weak Consequence. For all M, and s: *if* $[\![\varphi_1]\!]^+_s$ *and... and* $[\![\varphi_n]\!]^+_s$, *then not*$[\![\varphi]\!]^-_s$.

This particular choice would give us ordinary classical tautological inference (cf. van Benthem 1984).

The logic of Strong Consequence will be determined in section 3.3 below. As was noted already, fundamental classical principles such as

Excluded Middle or Non-Contradiction fail, because they may remain "undefined" due to lack of information. (But, contradictions $\varphi \wedge \neg\varphi$ still strongly imply everything ("Ex Falso Sequitur Quodlibet")—a principle only invalidated in the earlier-mentioned 4-valued approach. For no situation can make contradictions true, at least on our present analysis.) On the other hand, the classical "negative's travels" are all valid: $\neg\neg\varphi$ being strongly equivalent to φ, $\neg(\varphi \vee \psi)$ to $\neg\varphi \wedge \neg\psi$ and $\neg(\varphi \wedge \psi)$ to $\neg\varphi \vee \neg\psi$. (The reason is, essentially, that on the truth values 0 and 1, our connectives still behave classically: The present semantics is "conservative".) Notice that the latter principle is suspect in so-called "constructive logic"; a partial perspective is not necessarily a constructive one!

Nevertheless, the logic also has some curious surprises. For instance, Contraposition fails: φ may strongly imply ψ, without $\neg\psi$ strongly implying $\neg\varphi$. (A counter-example is $\varphi = p \wedge \neg p, \psi = q$.)

The reader may well feel bewildered by this abundance of notions of consequence, and correspondingly different logics. But then, this is the price of adopting a partial perspective: life just is not as simple as it might have seemed from earlier smooth formalizations. On the other hand, one may come to enjoy the present increased semantic sensitivity.

One well-known example in the early literature on situation semantics is the desired non-synonymy of p and $p \wedge (q \vee \neg q)$ in direct perception reports. And indeed, there is no strong equivalence between the two: $p \wedge (q \vee \neg q)$ strongly implies p, but not vice versa. Notice, however, that the two are "negatively synonymous," in the sense that, whenever p is definitely false, so is $p \wedge (q \vee \neg q)$ and vice versa.

So, we can distinguish various kinds of synonymy (and non-synonymy). Some expressions share the same 1-situations, but not 0-situations: an example is the pair $p \wedge \neg p$, $q \wedge \neg q$. Others show the converse behavior: An example is the dual pair $p \vee \neg p, q \vee \neg q$. And of course, there are very strongly equivalent pairs such as those in the above list of negation-interchange principles. Notice, by the way, that all necessary information about these notions is contained in our logic of Strong Consequence (if necessary, by taking suitably negated formulas).

Weak Negation

One recurrent theme in studies of partial logic is the feeling that, once bivalence has been abandoned, "negation" is no longer a univocal concept. There is a "cautious" version, with the truth table presented as before, but also a more sweeping version, requiring mere absence of truth:

		$\sim$
φ	1	0
	0	1
	u	1

Or, expressed in the alternative notation

$$\begin{array}{llll}(\sim) & [\![\sim\varphi]\!]_s^+ & \text{iff} & not{:}[\![\varphi]\!]_s^+ \\ & [\![\sim\varphi]\!]_s^- & \text{iff} & [\![\varphi]\!]_s^+\end{array}$$

Both types of negation have been used in the study of presupposition in natural language. The second negation would correspond to the NO-answer, when being asked the old question if (unmarried) John still beats his wife. Also, in ordinary discourse, both types of negation may be present. The first may occur in the syntactic vicinity of atomic predicates: "is running" versus "isn't running" might indeed be a non-exhaustive pair—whereas sentence negation at a higher level ("not every ship survived the Murmansk run") is often felt to be classically bivalent (see Seuren 1984).

Whether conclusive or not, these considerations certainly warrant an additional study of the above negation as well. And there are also honorable technical reason for including it. Notably, the language gains a greater power of expression, which will make the logical study easier. It also starts exhibiting new semantic features, notably *non-persistence*, which have to be faced eventually in any case, witness the recent interest in "non-monotonic reasoning."

As for expressive power, one may now define a useful "material implication" as follows:

$$\varphi \supset \psi =_{def} \sim \varphi \vee \psi$$

The resulting truth table becomes:

			ψ	
	$\supset$	1	0	u
φ	1	1	0	u
	0	1	1	1
	u	1	1	1

With this notion, a "Deduction Theorem" can be formulated for strong consequence:

$$\varphi \models_+ \psi \quad \text{iff} \quad \models_+ \varphi \supset \psi$$

This is in contrast with the earlier language, where no such result obtains. (In particular, $\neg\varphi \vee \psi$ does not do the job: E.g., $p \models_+ p$ without $\models_+ \neg p \vee p$.) Such a reduction of consequences with premises to pure validities is very useful in doing logical meta-theory.

Also, the new language has a more interesting persistence behavior. For instance, the earlier formulas are still both 1- and 0-persistent. But, e.g., $\sim p$ is only 0-persistent: when it is false (i.e., p is true), it remains so under extensions—whereas its truth has no such guarantee (in case p is undefined). On the other hand, $\sim\sim p$ is only 1-persistent—as is easily checked. Finally, $\sim p \wedge \sim\sim q$ is neither 0- nor 1-persistent.

To increase acquaintance with this language, here is a truth table list of all its non-strongly equivalent unary propositional functions in $\neg, \sim$:

φ	$\neg\varphi$	$\sim\varphi$	$\neg\sim\varphi$	$\sim\neg\varphi$	$\neg\sim\neg\varphi$
1	0	0	1	1	0
0	1	1	0	0	1
u	u	1	0	1	0

Moreover, the reader is referred to the *Normal Form* Lemma proved later on in section IV.3.

One interesting query is now to find a complete syntactic characterization of these various persistence classes. (This line of research was initiated in a more modal setting, in Veltman 1981.) We shall show that the only 1- and 0-persistent formulas are those of the original $\neg, \wedge, \vee$-language.

The logic of the present enriched language will be determined in section IV.3. Basically, $\sim$ satisfies all its laws from classical logic. Moreover, its interplay with $\neg$ obeys the strong implication

$$\neg\varphi \models_+ \sim\varphi$$

as well as the strong equivalence of φ and $\neg\sim\varphi$ (cf. the above truth tables). Notice however, that not even $\models_+ \sim\varphi$ implies $\models_+ \neg\varphi$: a counter-example being $\varphi = p \wedge \neg p$.

A Modal Perspective

The picture of "growing" situations, ordered by inclusion ($\sqsubseteq$) suggests adding a modality $\Box\varphi$ ("φ in all extensions"), $\Diamond\varphi$ ("φ in some extensions"). For instance, in such a language, persistence of a formula φ amounts to universal validity of $\varphi \supset \Box\varphi$. Also, the earlier notion of involvement becomes definable, using the formula $(\varphi_1 \wedge \ldots \wedge \varphi_n) \supset \Diamond\psi$. As usual in modal logic, validities obtained would reflect certain structural decisions concerning $< S, \sqsubseteq >$. E.g., the earlier-mentioned "convergence" (section IV.1) would show up via the axiom $\Diamond\Box\varphi \supset \Box\Diamond\varphi$. Notice, however, that no "genuine", contradictory alternative branching is permitted (yet) in this semantics, whence, e.g., the following is valid too: $\Diamond\varphi \supset \Box\sim\neg\varphi$.

But a more global ("S5-like") modal perspective is also possible, leading to a "meta-morphosis" of Strong Consequence.

There may be reasons for drawing "local" strong consequence into the formal object language studied here. Strong consequences true in a model express uniformities which themselves can be mentioned in the language. Thus, we shall also consider a language having an additional binary propositional operator $\Rightarrow$ with the stipulation that in a model M,

$$[\![\varphi \Rightarrow \psi]\!]^+ \text{ if, } \textit{for all } s \in S, [\![\varphi]\!]^+_s, \textit{ only if } [\![\psi]\!]^+_s$$

$$[\![\varphi \Rightarrow \psi]\!]^- \text{ if, } \textit{for some } s \in S, [\![\varphi]\!]^+_s, \textit{ without } [\![\psi]\!]^+_s$$

This stipulation makes $\Rightarrow$ subject to the law of Excluded Middle, and hence the pure $\neg,\wedge,\vee$-logic of such implication statements and their Boolean compounds is entirely classical. The situation would have been different had we stipulated, e.g.,

$$[\![\varphi \Rightarrow \psi]\!]^- \text{ if, } \textit{for some } s \in S, [\![\varphi]\!]^+_s \textit{ and } [\![\psi]\!]^-_s$$

Moreover, changes would also be needed if one wanted to incorporate the situation semantics idea that only some out of all actually valid uniformities may be "inside" the model. Such issues are not pursued here.

The above implication has a modal flavor, expressing a global entailment between propositions in the model. (Thus, it is not to be confused with the logical implication $\supset$.) As in modal logic, it turns out useful to introduce an explicit corresponding modality as well, for which we shall now, and henceforth, reserve the above "necessity" symbol $\Box$.

$$[\![\Box\varphi]\!]^+ \text{ if, } \textit{for all } s \in S, [\![\varphi]\!]^+_s$$

$[\![\Box\varphi]\!]^-$ if, *for some* $s \in S$,*not*$[\![\varphi]\!]^+_s$

The two notions are inter-definable:

$$\varphi \Rightarrow \psi \text{ and } \Box(\varphi \supset \psi) \text{ are strongly equivalent}$$

and so are

$$\Box\varphi \text{ and } (\varphi \supset \varphi) \Rightarrow \varphi$$

The resulting modal logic will be axiomatized in section IV.3.

2.2 Predicate logic

Having explored the propositional case in some detail, we can now pass on to richer languages on the same pattern of presentation.

In a first step, quantifiers are added to obtain an ordinary first-order predicate language. This time, the component D of the structures will come into play as well, being the (maximal) domain of individuals quantified over. As in the traditional approach in intensional logic, there is a choice to be made, whether to read the quantifiers "locally" (involving only individuals occurring in facts within the situation described) or "globally": i.e., ranging over the whole domain D. (Cf. Kamp 1983, for a discussion of this issue in the case of "see".)

Fortunately, as logicians, we need not take a stand here; but could develop both alternatives (cf. Langholm 1983). Here we shall only give the global reading, with the usual assignment convention:

$[\![\forall x\varphi]\!]^+_{s,A}$ if, *for all* $d \in D$, $[\![\varphi]\!]^+_{s,A^x_d}$

$[\![\forall x\varphi]\!]^-_{s,A}$ if, *for some* $d \in D$, $[\![\varphi]\!]^-_{s,A^x_d}$

Here, A, is an assignment to the free variables, and A^x_d is the assignment obtained from A by fixing the assignment of x to $d \in D$. Likewise,

$[\![\exists x\varphi]\!]^+_{s,A}$ if, *for some* $d \in D$, $[\![\varphi]\!]^+_{s,A^x_d}$

$[\![\exists x\varphi]\!]^-_{s,A}$ if, *for all* $d \in D$, $[\![\varphi]\!]^-_{s,A^x_d}$

Additional local readings can still be obtained, however, by considering suitable *restrictions*. For instance, once the restricted statement "all P are Q" is considered, its logical transcription $\forall x(Px \supset Qx)$ will work out to mean: "All things with property P *in our situation* have property Q".

Certainly, in natural language, quantifiers always occur relativized in some way—and hence the above distinction was a purely logical worry in any case. In fact, the determiner expression has a constant meaning throughout (compare Chapter III), with all "dynamic" aspects of reference to changing situations being handled by indications on their following restricting nouns.

But then the question arises if the predicate-logical formalism chosen should not mirror these features of natural language more closely. Indeed, the above reduction of restricted quantifiers to absolute ones using material implication (due to Frege) is known to break down for other quantities anyway (cf. Barwise and Cooper 1981 on the subject of "most").

In response, we could also consider two variants of the above predicate-logic. First, restricted quantifiers may be introduced using subscripted variables, the way mathematicians (and many off-duty logicians) often do. For instance "all doors creak" would become $\forall x_{door} : creak(x)$. This practice could be restricted to atomic predicates, forcing us to read, say, "all doors with a handle creak" as

$$\forall x_{door}(with\ a\ handle(x) \supset creak(x)).$$

Thus, a distinction is mirrored that was made earlier in our situation schemata, between leading conditions (given by the head common noun) and "conjoined" conditions (superimposed by the following restrictive relative clause).

The interpretation of restricted formulas is straightforward. For instance,

$$[\![\forall x_P \varphi]\!]^+_{s,A} \text{ if, } \textit{for all } d \in D \textit{ such that } [\![Px]\!]^+_{s,A^x_d},\ \ [\![\varphi]\!]^+_{s,A^x_d}$$

$$[\![\forall x_P \varphi]\!]^-_{s,A} \text{ if, } \textit{for some } d \in D \textit{ such that } [\![Px]\!]^+_{s,A^x_d}, [\![\varphi]\!]^-_{s,A^x_d}$$

Moreover, a modest "loading" mechanism could be implemented, mirroring the natural language possibility of letting common nouns range over one out of a choice of available situations (discourse situation, situation described, or yet other ones introduced in the process of interpretation). Technically, an operator *LOAD* can be added, forming new predicates out of atomic ones, whose semantic function is to "reset", in this limited case, the interpretation to the discourse situation or, as is also possible to implement, to a contextually given "resource situation":

$$[\![\, LOAD(P)x]\!]^+_{s,A} \text{ if } [\![Px]\!]^+_{d,A}$$

$$[\![\, LOAD(P)x]\!]^-_{s,A} \text{ if } [\![Px]\!]^-_{d,A}$$

This addition also marks the first occasion where the discourse situation explicitly enters into our semantic schema. One further way to exploit this would be to introduce explicit indexical expressions into the predicate-logical language, such as "I" or "you." In the following step, a different use will be made: the discourse situation is going to provide *temporal* reference points

As with the propositional language, many questions arise concerning (inter)definability of notions (i.e., expressive power), as well as semantic phenomena such as persistence—and, of course, the "mother question": axiomatization of the logic. Some samples of each will conclude this subsection.

As for definability, there are some "classical" equivalences. Notably, $\neg\exists x\varphi$ is equivalent to $\forall x\neg\varphi$, in the strong sense of sharing the same 1- and 0-situations, and the same holds for $\neg\forall x\varphi$ and $\exists x\neg\varphi$. As a consequence, $\neg\exists x\neg\varphi$ is equivalent strongly to $\forall x\varphi$ and $\neg\forall x\neg\varphi$ to $\exists x\varphi$. With weak negation the situation is similar. Notice, however, that differences remain when comparing the two. For instance, $\forall x \sim \varphi$ is strongly equivalent to $\neg\exists x\neg \sim \varphi$; but the latter is not strongly equivalent to $\neg\exists x\varphi$.

An interesting additional feature of the predicate-logical case arises with the earlier "meta-notion" $\Rightarrow$. Now, uniformities can be "parameterized", as in $\forall x\exists y(\varphi(x) \Rightarrow \psi(y))$. Indeed, this parameterized case is the standard form of situation semantics constraints.

Persistence remains the same notion in this setting. This time, the effect of our "global" stipulation for the quantifiers is such that all formulas in $\neg, \wedge, \vee, \forall, \exists$ are guaranteed to be persistent. With local readings, restricted to individuals actually occurring in the situation described, existential quantification would remain safe, whereas universal statements would typically run the risk of invalidation in a wider setting. This standard model-theoretic observation shows up with our restricted formulas: neither $\forall x_p Qx$ nor $\forall x(Px \supset Qx)$ is persistent.

Also our additional semantic apparatus generates new questions of its own. For instance, the *LOAD*-mechanism has some of the features of the well-known operation of Relativization to certain subdomains. One sample indication of this is expressed in the following simple observation, relating truth of formulas with "loaded" parts in a larger model to truth of those formulas in their "pure" form in suitably restricted situations. When all atomic predicates in φ are loaded,

$$[\![\varphi]\!]_s^+ \text{ if and only if } [\![\ UNLOAD(\varphi)]\!]_d^+$$

$$[\![\varphi]\!]_s^- \text{ if and only if } [\![\ UNLOAD(\varphi)]\!]_d^-$$

Here, $UNLOAD(\varphi)$ is the "unloaded" version of φ, i.e., φ stripped of all its "*LOAD*"'s.

Finally, as for the logic obtained, there are the standard classical rules for the quantifiers $\forall, \exists$. Together with the definition of relativized quantifiers, and our implementation of the loading mechanism, these imply the expected rules for the latter. Some illustrations may be found in section IV.3.

2.3 Tense logic

2.3.1 Introduction. Up till now, statements in our languages mainly expressed types of situations without any relation to an actual vantage point in the utterance. One of the most pervasive anchoring mechanisms of the latter kind is the tense marking found in natural languages. Tenses relate situations described to the "temporary" present of the utterance. Of course, natural language has a much richer system of temporal indicators than just tense—including temporal auxiliaries ("have," "will"), adverbs ("always," "yesterday"), prepositional phrases ("for an hour"). We shall only scratch the surface of this mechanism here, which neatly illustrates the truly *relational* character of meaning.

There is a long tradition of logical interest in tenses and related constructions, with pioneers such as Reichenbach (1947) and Prior (1967)—and even a technical flowering in the sixties and seventies. Here, only a few issues are outlined, to provide some background for our own proposals. (A fuller story is found in van Benthem (1982).)

Perhaps the dominant approach in "tense logic" has been that of Prior, formalizing tenses as ***propositional operators***, *PAST*, *FUTURE*, etc., in a formal language interpreted pointwise in temporal orders $< T, \prec >$. Typically, a semantic clause for the past tense would read like this:

$$M \models PAST\varphi[t] \quad \text{iff} \quad \textit{for some } t' \prec t, M \models \varphi[t']$$

Thus, tenses are explained using (existential) quantification over points in time.

Starting from this, further tenses have been treated by introducing more complex formulas on the right-hand side. Also, temporal indexicals, such as "now" or "then", were incorporated, leading to an extension of the framework with further contextual reference points in addition to "the point described". Thus, on the account given in Kamp 1971, "now" resets evaluation to the moment of utterance:

$$M, t_o \models NOW\varphi[t] \quad \text{iff} \quad M, t_0 \models \varphi[t_0]$$

Moreover, various combinations of time and modality have been treated–two notions that are entangled in many ways. These exhibit the familiar scheme (here without indexicality) of truth in a world at a time:

$$M \models \varphi[w, t]$$

(A good survey is Thomason 1984.)

Finally, more on the ontological or mathematical side of the enterprise, the point perspective has been challenged, in favor of an "interval semantics" employing basic temporal items that are extended. (See Kamp 1979 for the combined linguistic and philosophical motivation behind this move.) On this view, the temporal structures are rather of the form $< I, \prec, \sqsubseteq, \circ >$, where relations of *inclusion* ($\sqsubseteq$) and *overlap* ($\circ$) testify to the extended nature of intervals. The latter have been invoked, not just to obtain a more adequate modeling of tense, but also of natural language *aspect*, encoding various types of duration and termination of events.

Tense logic has known a certain prosperity, as a tool for analyzing arguments involving time (recently also, as a vehicle for formalizing reasoning about execution of computer programs). But, as a mirror of natural language tense, it always seemed a bit out of focus. For instance, match-ups between Priorean operators and actual tenses never quite succeeded. And indeed, the very quantifier reading given above for the simple past is endangered by counter-examples, such as Partee's famous "I didn't turn off the stove"; which seems to mean neither *PAST NOT* nor *NOT PAST*: the only two Priorean possibilities. Rather, some past period seems to be given contextually, and my not turning off the stove refers to that. In other words, the past tense can have a "deictic" character, referring to some specific past episode.

Actually, this issue is a rather delicate one. There are certain phenomena concerning tense which still seem to favor the quantifier explication, such as the scope ambiguity in "Every man present today was born on a Sunday". A proposal which does justice to both types of observation would be one which keeps the existential quantifier reading, but treats it ("specifically") in the non-generalized quantifier way of Chapter III, introducing a reference marker which can be accessed by the discourse situation. But certainly, the deictic point of view by itself deserves logical exploration.

2.3.2 Tense and time in situation semantics. The treatment of tense in *Situation and Attitudes* is deictic, witness our account in Chapter III. In fact, only two tenses are treated: *present* and *past*; but there are also some proposals concerning the auxiliaries "have" and "will." We shall introduce formal languages reflecting these proposals below. But first, some general features of the approach should be discussed.

Simplifying a bit, the semantic schema will now become, e.g.,

$$\text{in } M : {}_{d.c}[\![PAST\varphi]\!]_{s,l} \quad \text{iff} \quad {}_{d,c}[\![\varphi]\!]_{s,c(PAST)} \ \textit{and}\ c(PAST) \prec l_d$$

Thus, the tense marker gets assigned some temporal item before the discourse time. In other words, what used to be an *existential unary property* of the situation described (with its accompanying location), now becomes a *binary relation*, of which the former property was a "right projection." This move is one that recurs in situation semantics: a richer setting of evaluation makes additional "parameters" explicit, while simplifying the truth conditions. (On the other hand, of course, the burden imposed on the context is increased.)

Notice that the above scheme also presupposes a rich kind of situations s, which have something to say about various locations. In other words, we must think of it as a *course of events* (i.e., a partial history—"partial" both in the sense of not necessarily encompassing *all* locations and also of not necessarily exhausting the truth about each of its locations). This is quite a move from the earlier notion of a "graspable" local situation that we find ourselves in: the latter would have only one location, or just a handful.

On this view, however, an explication of the meaning of the past tense becomes more problematic—as one should have to specify *which* situation at *c(PAST)* was intended (if s itself does not live there). That a context would supply all this, seems rather implausible—so one might have to quantify after all ("some situation on *c(PAST)*"). There are various pitfalls with this latter tack, not to be pursued here. Thus, the innocent-looking semantic clause for tense actually forces one to reconsider the intuitive idea behind the very notion of a "situation".

Another set of deep questions concerns the background ontology chosen here. As was explained in Chapter III, the location structure of *Situations and Attitudes* has relations of precedence ($\prec$), and temporal overlap ($\circ$). The second is used in the truth condition for the present tense:

$$_{d,c}[\![PRESENT\varphi]\!]_{s,l} \quad \text{iff} \quad _{d,c}[\![\varphi]\!]_{s,c(PRESENT)} \; and \; c(PRESENT) \circ l_d$$

This explication of the present tense is certainly debatable. Does a tiny temporal overlap with the discourse time make an event present? At least various equally plausible alternatives exist, such as temporal simultaneity or, at most, inclusion. Indeed, simply requiring *c(PRESENT)* to be identical with the discourse location itself seems to be a strong contender. As usual, in our perspective, these are all possible topics of investigation.

In a semantical theory deriving its models from cues within natural language, such options are significant—as our view of the relevant location structure will be influenced by the outcome of the above discussion. But one could also take a more absolute ontological point of view, philosophizing about what is the correct space-time structure to be invested in semantics. In line with the current "real world" ideology of situation semantics, there might even be just one unique real space-time.

Even if there is such a unique structure Λ, it certainly manages to hide itself from us very effectively. There is no philosophical consensus about what are the basic primitive relations in space-time—and intuitions concerning axioms governing them are indeterminate, and not always consistent. For instance, one large issue is whether to think of locations as already coming with temporal and spatial parts, say, as regions in a classical Newtonian space-time—or as primary entities, from which time and space must arise through some process of abstraction. In the latter case, a classical physical outcome with the above convenient separation is not guaranteed.

To settle upon at least one definite picture, one may let Λ consist of connected regions in classical space-time, i.e., ordinary Euclidean space married to linear time. For basic primitives, we will interpret the above precedence as "precedence of temporal content", and the above temporal overlap as "overlap of temporal content". These two notions satisfy the following postulates (cf. Kamp 1979):

(1) $\forall x : \neg x \prec x$ (irreflexivity)
(2) $\forall xyz : x \prec y \wedge y \prec z \rightarrow x \prec z$ (transitivity)
(3) $\forall x : x \circ x$ (reflexivity)
(4) $\forall xy : x \circ y \rightarrow y \circ x$ (symmetry)
(5) $\forall xy : x \circ y \rightarrow \neg(x \prec y)$
(6) $\forall xyzu : x \prec y \wedge y \circ z \wedge z \prec u \rightarrow x \prec u$
(7) $\forall xy : x \circ y \vee x \prec y \vee y \prec x$ (linearity)

Actually, (1) and (2) follow from the remainder. A very crisp re-axiomatization should be mentioned as well, due to Thomason (1983):

(i) $\forall x : \neg x \prec y$
(ii) $\forall xyzu : x \prec y \wedge z \prec u \rightarrow x \prec u \vee z \prec y$
(iii) $\forall xy : x \circ y \leftrightarrow \neg x \prec y \wedge \neg y \prec x$

These postulates will be used below. Notice that all of them have a universal form, expressing general constraints on whatever number of locations is present. Existential principles expressing the actual richness of space-time locations would require additional considerations of a different kind.

It is well-known that the above Kamp postulates suffice for making a linear time structure out of the location sets, by any one of a number of familiar methods going back to Russell and Whitehead.

One final relation to be mentioned here is *temporal inclusion*, in the sense of "inclusions of temporal content." This notion plays a role in stating "hereditary" properties of certain types of event description in natural language (see below). An interval ontology based upon precedence and inclusion may be found in van Benthem (1982).

More purely *spatio-temporal* relations, and indeed the spatial aspect in general, will not be considered when interpreting the following formal languages.

2.3.3 *Two tenses.* To mirror this view of tense, our formal language (propositional or quantificational) may be enriched with two propositional operators *NOW* and *THEN*. Their meaning relations are now straightforward:

$$_{d,c}[\![NOW\varphi]\!]^{+}_{s,l} \quad \text{iff} \quad _{d,c}[\![\varphi]\!]^{+}_{s,c(NOW)} \; \mathit{and} \; c(NOW) \circ l_d$$

$$_{d,c}[\![NOW\varphi]\!]^{-}_{s,l} \quad \text{iff} \quad _{d,c}[\![\varphi]\!]^{-}_{s,c(NOW)} \; and \; c(NOW) \circ l_d$$

$$_{d,c}[\![THEN\varphi]\!]^{+}_{s,l} \quad \text{iff} \quad _{d,c}[\![\varphi]\!]^{+}_{s,c(THEN)} \; and \; c(THEN) \prec l_d$$

$$_{d,c}[\![THEN\varphi]\!]^{-}_{s,l} \quad \text{iff} \quad _{d,c}[\![\varphi]\!]^{-}_{s,c(THEN)} \; and \; c(THEN) \prec l_d$$

Proposals in this vein have been around in the literature for a while.

Although we can iterate *NOW* and *THEN* in our language, evaluation will always reduce to the location assigned to the innermost of these, the others are irrelevant. This is as it should be—natural language has no iterated tenses. This is not to say that the correspondence is perfect, however. As was indicated in Section 2, different occurrences of tense may have different contextually indicated locations, sometimes even within the same sentence.

The above semantic clauses reflect a change of presentation which deserves emphasis. Our statements concerning individuals are now thought of as evaluated in a situation s *at* a certain location l. And tense operators give us explicit instructions about which locations l are involved, according to the discourse connection c.

This dependence on *extended* locations, a central point in situation semantics, suggests new questions concerning *temporal preservation* behavior. For instance, for which types of statements φ will truth "in s at l" guarantee truth in s at all sublocations $l' \sqsubseteq l$ (insofar as the latter are present in s)? Expressions in natural language fall into various broad classes of semantic behavior in these respects, witness the well-known Vendler Classification (into 'states', 'activities', 'accomplishments' and 'achievements'). Such questions are being considered in the earlier-mentioned "interval semantics" for tense logic. They ought to be just as relevant here. Indeed, eventually, one would hope for an enlightening semantic classification replacing Vendler's empirical one.

In connection with this temporal "heredity" towards subintervals, another potential bonus of a partial approach appears. In many discussions in this area, it has been observed that we can view the world at different levels of temporal "grain". The smallest units at one level may become composites at another. Now, one striking feature in all this is that language appropriate at one level need not be appropriate at another (finer or coarser) level. This awkward feature for traditional approaches is no problem whatsoever in situation semantics: several "courses of events" may describe one history at different levels of temporal detail, possibly employing quite different relations.

The logic of the above scheme is rather unexciting. For instance *NOW* merely interchanges with the other operators:

$$NOW(\varphi \wedge \psi) \leftrightarrow NOW\varphi \wedge NOW\psi$$

$$NOW(\varphi \vee \psi) \leftrightarrow NOW\varphi \vee NOW\psi$$

$$NOW\exists x\varphi \leftrightarrow \exists x\ NOW\varphi$$

$$NOW\forall x\varphi \leftrightarrow \forall x\ NOW\varphi$$

(This principle would not be valid with the *local* reading of the quantifiers discussed above. Cf. Kamp 1971 for a comparison with a more classical approach.) Negation presents the only interest, in that a little care is needed:

$$NOW\neg\varphi \leftrightarrow \neg\ NOW\varphi$$

$$NOW\sim\varphi \leftrightarrow\sim NOW\varphi \wedge\ NOW\ \textbf{\textit{true}}$$

where ***true*** is some universal validity, say $(p \supset p)$. (The conjunct '*NOW* ***true***' has been added to express that $c(NOW)$ overlaps the discourse location.

Finally, as was observed before, sequences of operators collapse,

$$NOW\ NOW\varphi \leftrightarrow NOW\varphi$$

We conjecture that this list forms the complete logic of *NOW* in the present setting. Analogous principles hold for *THEN*.

Notice that the structure of the location set Λ played no role at all in this logic: *NOW* and *THEN* merely reflect the mechanics of deixis.

2.3.4 Two temporal auxiliaries. In traditional tense logic, a past tense statement would be formalized as $PAST\varphi$ ("Pluto kicked back"). Then, an iterated formula such as $PAST\ PAST\varphi$ would be taken to express the past perfect: "Pluto had kicked back". Linguists have frequently observed that morphology and syntax ought to be taken more seriously here. The second sentence is still a simple past, and that of a form "–have kicked back," which shows an independent syntactic existence: "To have kicked back was Pluto's finest act". Thus a more sensitive semantic account seems preferable, reflecting this distinction between tense and temporal auxiliaries—even if the borderline is not the same in all languages.

Interestingly, it is precisely in this perspective that some central features of the Priorean approach can be endorsed after all. (Compare the suggestions made in *Situations and Attitudes*, pp. 288-289.) Let us enrich our language with two more operators, reflecting the perfect and future auxiliaries:

$$[\![HAVE\varphi]\!]^{+}_{s,l} \text{ iff } \textit{for some } l' \prec l, [\![\varphi]\!]^{+}_{s,l'}$$

$$[\![HAVE\varphi]\!]^{-}_{s,l} \text{ iff } \textit{for all } l' \prec l, [\![\varphi]\!]^{-}_{s,l'}$$

and dually toward the future:

$$[\![WILL\varphi]\!]^{+}_{s,l} \text{ iff } \textit{for some } l' \succ l, [\![\varphi]\!]^{+}_{s,l'}$$

$$[\![WILL\varphi]\!]^{-}_{s,l} \text{ iff } \textit{for all } l' \succ l, [\![\varphi]\!]^{-}_{s,l'}$$

These stipulations are only meant as examples. Especially, the negative clauses admit of alternatives. For instance, on the account given here, a course of events s must be extremely rich to refute a perfective statement. A more generous alternative would be to require only "*not* $[\![\varphi]\!]^{+}_{s,l'}$", a more parochial one to only consider past locations on which s is defined.

In any case, with this type of meaning assigned, the relational structure of Λ does become relevant to the logic, even crucially so. Various decisions concerning conditions on precedence will show up directly in logical validities of the language. This mutual influence was a striking one in traditional tense logic—and indeed, a direct interplay between model-theoretic structure and (natural) language inference is one of the more exciting phenomena in semantics.

The interpretation scheme employed above will already validate a certain minimal logic, containing principles such as

$$HAVE(\varphi \vee \psi) \leftrightarrow HAVE\varphi \vee HAVE\psi$$

An axiomatization of this minimal logic will not be provided, adding this question to our already long list of open problems and suggestions for further research.

Richer structural conditions on temporal precedence, such as (1)-(7) or (i)-(iii) above, may induce further logical validities. For instance, irreflexivity has no immediate effects in the present formal language, but transitivity does. Its truth amounts to validity of the principle

$$HAVE\ HAVE\varphi \supset HAVE\varphi$$

And the much stronger Thomason axiom (ii) also has its reflections, such as e.g.:

$$HAVE(\varphi \wedge HAVE\psi) \wedge HAVE(\alpha \wedge HAVE\beta) \supset$$
$$HAVE(\alpha \wedge HAVE\psi) \vee HAVE(\varphi \wedge HAVE\beta)$$

A task of possibly greater difficulty is to axiomatize the *HAVE/WILL*-logic of one single mathematical temporal structure, the open intervals on the real number line. Given the recent situation semantics commitment to single "standard models", this type of query might even be the more fundamental one. For an answer in a more traditional tense logical setting, see the ingenious paper by Burgess 1982.

Finally, it would be interesting to treat further linguistic constructions involving temporal *overlap* and *inclusion* structure as well, with reflections of the latter in the logic. For instance, one, rather simple-minded reading of the progressive would naturally involve inclusion:

$$[\![BE\varphi{-}ING]\!]^{+}_{s,l} \text{ iff } \textit{for some } l' \sqsupseteq l, [\![\varphi]\!]^{+}_{s,l'}$$

In this case, suitable inclusion postulates would have to be imposed on Λ (cf. van Benthem 1982). Then, what is the logic of *HAVE, WILL*, and *BE-ING*?

2.4 Temporal predicate logic

The logical formalisms of the preceding sections had operators corresponding relatively closely to natural language tense markers and temporal auxiliaries. The price to be paid for this was a certain tension between principles expressible in these formalisms and the most natural way of formulating conditions on the D, Λ-structure in our models—viz. by means of first-order expressions mentioning and quantifying over locations explicitly. But then, this tension also resulted in interesting logical questions concerning reflection of the latter in the logic of the former. Moreover, insofar as the operator format is closer to actual linguistic constructions, a direct description of its logic will presumably tell us more about the peculiarities of inference expressed in the medium of natural language than a "smoother", but also more remote logical formalism.

There has been an endemic debate in traditional tense logic, whether to stick with Prior-like formalisms, or to adopt an explicitly "temporally parameterized" predicate logic. Arguments have involved issues of logical convenience, philosophical parsimony and linguistic integrity. (An interesting, though partisan review is Needham 1975.) As an illustration of the concrete difference in representation, the choice would be between formulas such as

$$NOW\ WILL \forall x (NOW\ HAVE\ Px \supset Qx)$$

or

$$\exists t \succ t_0 \forall x (\exists t' \prec t_0 \wedge P_{t'} x \supset Q_t x)$$

In the long run, the latter wins in terms of perspicuity.

We do not aim to settle old scores, but to explore interesting alternatives. We shall study a *two-sorted* predicate logic in section IV.3, with the possibility of "localized" atomic predicates, $P_l x_1 \dots x_n$, as well as quantification over locations. Moreover, the basic temporal relations $\prec$ and $\circ$ will be represented by constants in the language.

In the most general case, one might envisage both *located* and *unlocated* predicates. Moreover, dependence might be on one or on several locations. A traditional example of the latter phenomenon is Russell's "I thought your yacht was longer than it is", involving a relation "x is longer at t_1, than y at t_2." In particular, "localizing" special predicates, such as identity or precedence raises some interesting issues. For instance, could a precedence statement $l_1 \prec l_2$ itself be relative to a vantage point l? The latter relativistic possibility has not been implemented here: facts about Λ will be treated as being absolute, as was already apparent in the truth conditions of Chapter III.

The logic of this two-sorted language will be determined in section IV.3 below, on the analogy of the purely predicate-logical case. In a sense, the task has now become much easier than in the tense-logical case above, as structural conditions on Λ can be directly recorded. Nevertheless, the explicit display of locations also has its pleasant side-effects. For instance, in a language such as this inferences can involve the discourse location itself. I.e., the phenomenon of "inverse information" from situation described to utterance situation, stressed in situation semantics, shows up inside a formal language for the first time.

2.5 *Situated temporal predicate logic*

Once on the road of bringing out semantic structure within the linguistic formalism, the next obvious candidate are the *situations* themselves. Thus, we could also consider a three-sorted first-order language, allowing quantification over situations, and representing at least their basic relation of *inclusion*. As compared with the preceding completeness theorem, no new features emerge—unless we start making very specific assumptions about situations. For instance, identifying the latter with *finite partial substructures* (or, sets of facts) from the global model M will even lose us axiomatizability (cf. Kamp 1984). Another logical theme recreating some "tension" might be the comparison between possible *higher-order* conditions on our situation structures and their (restricted) reflection in a *first-order* formalism.

But on the whole, we feel that, once a rich language like this is considered—being a highly expressive fragment of our former semantic metalanguage—the most urgent issue is not a logical but a philosophical or mathematical one. What is a reasonable "theory of situations"? In other words, we have come to the original perspective of a direct mathematical study of our situation structures or "algebras." As will have become apparent from various remarks throughout this part, the latter enterprise may have a great deal of intrinsic interest.

3 MATHEMATICAL STUDY OF SOME FORMAL LANGUAGES

In this section we introduce a two-sorted first order language which we will use as a vehicle for discussing a number of the mathematical and logical questions raised in earlier sections.

3.1 Definition of structure

We have to be a bit more precise in spelling out the details of our notion of *situation semantic structure*. We have four non-empty domains

S *situations*
D *individuals*
Λ *locations*
R *relations*

We assume no internal structure on the first two domains. Relations can be either located or unlocated and they come with a specific "arity". We thus take R to consist of two families $\{R^l_n\}_{n\geq 0}$ and $\{R^u_n\}_{n\geq 0}$, where e.g. R^l_2 is the set of binary located relations in our model.

The location part Λ has one internal structural relation, *precede*, i.e. Λ is of the form $\Lambda =< L, precede >$ where L is a non-empty set and *precede* is a binary relation on Λ satisfying the structural conditions

(i) $< \lambda, \lambda > \notin$ *precede*
(ii) $< \lambda_1, \lambda_2 >, < \lambda_3, \lambda_4 > \in$ *precede* implies that either
$< \lambda_1, \lambda_4 > \in$ *precede* or $< \lambda_3, \lambda_2 > \in$ *precede*

See the discussion of the Thomason axioms (i) – (iii) in section 3.2.3.; we have not included *overlap* as a primitive since it is definable in terms of *precede*, cf. axiom (iii).

The four domains are connected by the "global" relation:

$$in\ s : \text{at } \lambda : r, a_1, \ldots, a_n; pol$$

which in the model theory is represented by a set In of tuples

$$< s, \lambda, r, a_1, \ldots, a_n, pol >$$

where the location λ is optional. The set In will respect a certain *consistency constraint*: For any two situations s and t of S,

$$< s, \vec{\alpha}, 1 > \in In \ \text{ implies } \ < t, \vec{\alpha}, 0 > \notin In$$

(i.e. each situation is "actual") and a *compatibility constraint*

$$< s, \lambda, r, a_1, \ldots, a_n, pol > \in In \ \text{ implies } \ r \in R_n^l$$

$$< s, r, a_1, \ldots, a_n, pol > \in In \ \text{ implies } \ r \in R_n^u.$$

Many of the connectives and operators, such as $\sim$, $\neg$, $\wedge$, $\vee$, $\forall$, $\exists$, have the property of not "looking beyond" the situation in which the truth or falsity of some formula is being determined. Formulas built up from such connectives, are "situational first order" in the sense that no reference is made to situations, either directly or by some kind of quantification. The first language we shall introduce is first order in this sense. We are thus in a "single-situation" context and we restrict the notion of structure correspondingly. A structure in this restricted sense will be of the form

$$< D, R, \Lambda, in >,$$

where D,R,Λ are as before and in is a set of tuples $< (\lambda), r, a_1, ..a_n, pol >$, i.e. the λ is optional, satisfying a *consistency condition*

$$<\vec{\alpha}, 1 > \in in \ \text{ implies } <\vec{\alpha}, 0 > \notin in,$$

and a *compatibility condition* similar to the one for the full model structure.

Notice that we do not by this move deny the relational point of view; in these restricted models the discourse and the (possible) resource situations in a sense reside "outside" the model structure and provide the background for the interpretation of various constants of the language, cf. our discussion in 3.1.

If $< S, D, R, \Lambda, In >$ is a full model structure and $s \in S$, then $< D, R, \Lambda, in_s >$ is a single-situation model structure, where

$$<\vec{\alpha}>\in in_s \quad \text{iff} \quad < s, \vec{\alpha}>\in In$$

3.2 The system L_3

We introduce now the "situational first order" language mentioned above. It is basically a two-sorted language with the two negation symbols $\sim$ and $\neg$.

3.2.1 Formulas and interpretations. Formulas are built up from the following symbols:

- *Relation symbols:*
  For each $n \geq 0$ an infinite list of n-ary, *unlocated* relation symbols, and an infinite list of n-ary, *located* relation symbols.
- *Special relation symbol:* $\prec$ (for temporally precedes)
- Infinite lists of

  individual variables: $x_0, x_1, \ldots$

  individual constants: $a_0, a_1, \ldots$

  location variables: $\ell_0, \ell_1, \ldots$

  location constants: $l_0, l_1 \ldots$
- *Propositional constants:* t (truth), f (falsity)
- *Logical symbols:* $\neg, \sim, \wedge, \vee, \forall, \exists$
- *Auxiliary symbols:*) , (

The set of *formulas* is defined recursively as follows:

- t and f are formulas.

- If R is an n-ary unlocated relation symbol and $\alpha_1, \ldots, \alpha_n$ are individual variables or constants, then $R(\alpha_1, \ldots, \alpha_n)$ is a formula.
- If R is an n-ary located relation symbol, β a location variable or constant and $\alpha_1, \ldots, \alpha_n$ are individual variables or constants, then $R(\beta, \alpha_1, \ldots, \alpha_n)$ is a formula.
- If β_1 and β_2 are location variables or constants, then $(\beta_1 \prec \beta_2)$ is a formula.
- If φ and ψ are formulas, then so are $\neg\varphi$, $\sim \varphi$, $(\varphi \wedge \psi)$, and $(\varphi \vee \psi)$.
- If φ is a formula and α a variable (individual or location), then $\exists\alpha\varphi$ and $\forall\alpha\varphi$ are formulas.

In addition, it will be useful to allow $(\varphi \supset \psi)$ and $(\varphi \equiv \psi)$. These extended formulas will be considered as abbreviations of $(\sim \varphi \vee \psi)$ and $((\varphi \supset \psi) \wedge (\psi \supset \varphi))$ respectively.

This is our language, it is two–sorted, e.g.

$$\forall \ell((\ell \prec l) \supset \exists x R(\ell, x, a))$$

is a formula of our language.

We next turn to the topic of interpreting the language. A formula contains constants and variables; a *variable assignment* will handle the latter. To handle the former we introduce an *interpretation function* ζ which maps n-ary located [unlocated] relation symbols to n-ary, located [unlocated] relations, individual constants to individuals, and location constants to locations. By a model we shall mean the structure

$$M =< D, R, \Lambda, in, \zeta >,$$

where we have added ζ. As above we use

$$[\![\varphi]\!]^+_{M,A} \;,\; [\![\varphi]\!]^-_{M,A}$$

to mean that φ is true [resp. false] in the model M under the variable assignment A.

We now embark on the inductive definition of interpretation. To facilitate the exposition we let ξ be the function which is equal to ζ if applied to constants, A if applied to variables. The definition is standard, we indicate the main cases. For a located relation symbol P

$$[\![P(\beta, \alpha_1, \ldots, \alpha_n)]\!]^+_{M,A} \quad \text{iff} \quad < \xi(\beta), \zeta(P), \xi(\alpha_1), \ldots, \xi(\alpha_n), 1 > \in in_M$$

$$[\![P(\beta, \alpha_1, \ldots, \alpha_n)]\!]^-_{M,A} \quad \text{iff} \quad < \xi(\beta), \zeta(P), \xi(\alpha_1), \ldots, \xi(\alpha_n), 0 > \in in_M$$

A similar stipulation applies to unlocated relation symbols.

$$[\![(\beta_1 \prec \beta_2)]\!]^+_{M,A} \quad \text{iff} \quad < \xi(\beta_1), \xi(\beta_2) > \in \mathit{precede}$$

$$[\![(\beta_1 \prec \beta_2)]\!]^-_{M,A} \quad \text{iff} \quad \mathit{not}\ [\![(\beta_1 \prec \beta_2)]\!]^+_{M,A}$$

Hence the interpretation of $(\beta_1 \prec \beta_2)$ obeys a *completeness* requirement, as well as a *consistency* requirement. This will show up in the axiomatization. Finally,

$$[\![t]\!]^+_{M,A} \text{ and } \mathit{not}\ [\![t]\!]^-_{M,A}$$

$$[\![f]\!]^-_{M,A} \text{ and } \mathit{not}\ [\![f]\!]^+_{M,A}$$

The recursive rules for connectives and quantifiers we have already given in section IV.2.1.2; and (for quantification with non-indexed variables) section IV.2.2. The assumption

$$<\vec{\alpha}, 1 > \in in \quad \text{implies} \quad <\vec{\alpha}, 0 > \notin in$$

guarantees that no *atomic formula* is both true and false. A simple induction shows that no *formula* is both true and false.

If this constraint is left out we get a notion of *generalized model* with an associated four-valued logic, cf. section IV.2.1. This notion,which may have some independent interest, will be a useful technical tool in the completeness proof. Notice that the above completeness/consistency requirements for the precedence relation $\prec$ still hold in the generalized models.

Since generalized models lack the consistency constraint they are easier to work with from a mathematical point of view. And the relationship between the consistency constraint and the axiom $\neg\varphi \supset\sim \varphi$ (φ atomic) allows us to obtain corresponding results for proper models.

3.2.2 Axiom system. The system L_3 has the following *axiom schemas*:

A1. $(\varphi \vee \varphi) \supset \varphi$

A2. $\varphi \supset (\varphi \vee \psi)$

A3. $(\varphi \vee \psi) \supset (\psi \vee \varphi)$

A4. $(\varphi \supset \psi) \supset ((\chi \vee \varphi) \supset (\chi \vee \psi))$

A5. $(\varphi \wedge \psi) \supset \varphi$

A6. $(\varphi \wedge \psi) \supset (\psi \wedge \varphi)$

A7. $\varphi \supset (\psi \supset (\varphi \wedge \psi))$

A8. $\neg\varphi \supset \neg(\varphi \wedge \psi)$

A9. $\neg(\varphi \wedge \psi) \supset \neg(\psi \wedge \varphi)$

A10. $\neg(\varphi \vee \psi) \supset \neg\varphi$

A11. $\neg(\varphi \vee \psi) \supset \neg(\psi \vee \varphi)$

A12. $(\neg\varphi \wedge \neg\psi) \supset \neg(\varphi \vee \psi)$

A13. $\neg(\varphi \wedge \psi) \supset (\neg\varphi \vee \neg\psi)$

A14. $\varphi \supset \neg\neg\varphi$

A15. $\neg\neg\varphi \supset \varphi$

A16. $\varphi \supset \neg \sim \varphi$

A17a. $\neg \sim \varphi \supset \sim\sim \varphi$

A17b. $\neg\varphi \supset \sim \varphi$ $\qquad$ φ *atomic*

A18. t

A19. $\sim \neg t$

A20. $\neg f$

A21. $\sim f$

A22. $\sim (\beta_1 \prec \beta_2) \equiv \neg(\beta_1 \prec \beta_2)$

A23. $\sim (\beta \prec \beta)$

A24. $((\beta_1 \prec \beta_2) \wedge (\beta_3 \prec \beta_4)) \supset ((\beta_1 \prec \beta_4) \vee (\beta_3 \prec \beta_2))$

In the following group of axioms α is an individual or location variable, β a constant or variable of the appropriate sort. We assume that α does not occur within the scope of any quantifier binding β in A25 and A26:

A25. $\forall\alpha\varphi \supset \varphi(\beta/\alpha)$

A26. $\varphi(\beta/\alpha) \supset \exists\alpha\varphi$

A27. $\neg\forall\alpha\varphi \equiv \exists\alpha\neg\varphi$

A28. $\neg\exists\alpha\varphi \equiv \forall\alpha\neg\varphi$

We now come to the *rules of inference*.

R1:

$$\frac{\varphi, \varphi \supset \psi}{\psi}$$

R2: For α not free in χ:

$$\frac{\chi \supset \varphi}{\chi \supset \forall \alpha \varphi}$$

R3: For α not free in χ:

$$\frac{\varphi \supset \chi}{\exists \alpha \varphi \supset \chi}$$

We say that a formula is *valid* if it is satisfied in every model under every variable assignment. A *schema* is valid if every instance of it is valid. We first notice that all axioms are valid. Observe that the three connectives $\wedge, \vee, \sim$ cannot distinguish between falsity and non-truth; $\wedge, \vee, \sim$ lump 0 and u together as non-truth, and function exactly as in ordinary two-valued propositional calculus. Hence any ordinary tautology in the connectives $\wedge, \vee, \sim$ is valid in the extended three-valued, or even four-valued calculus ($\wedge$, $\vee$, $\sim$ similarly "lump" 1 and "overdefined" together as truth.), provided negation is weak negation.

The 0-ary unlocated relation symbols are essentially our propositional variables. A substitution instance is obtained by uniform substitution of formulas for propositional variables. Since no special semantical restrictions are posed on propositional variables, any substitution instance of a valid formula is valid. Hence A1-A7 which axiomatize ordinary propositional calculus are valid in the three- and four-valued logics we are considering.

The only axiom which separates the generalized models from the proper ones is axiom A17b.

Definition. L_4 is the axiom system without A17b.

We will first show that L_4 axiomatizes the generalized four-valued semantics; then show that A17b is sufficient to ensure consistency. But first we need some technical results about the system L_4.

3.2.3 Some lemmas and theorems.

Theorem (Positive Equivalence Theorem). *Let $L_4 \vdash \alpha \equiv \beta$ and let φ come from ψ by substitution of α for some occurrence of β not within the scope of any $\neg$. Then $L_4 \vdash \varphi \equiv \psi$.*

Proof: by induction

Basis: If ψ does not contain β or if ψ is β, the theorem follows trivially.

Induction: Since A1-A7 axiomatizes ordinary propositional calculus and the following formulas are tautologies:

$$(\varphi_1 \equiv \psi_1 \wedge \varphi_2 \equiv \psi_2) \supset ((\varphi_1 \wedge \varphi_2) \equiv (\psi_1 \wedge \psi_2))$$

$$(\varphi_1 \equiv \psi_1 \wedge \varphi_2 \equiv \psi_2) \supset ((\varphi_1 \vee \varphi_2) \equiv (\psi_1 \vee \psi_2))$$

$$(\varphi_0 \equiv \psi_0) \supset (\sim \varphi_0 \equiv \sim \psi_0),$$

the induction steps for $\wedge, \vee, \sim$ follow. There is no induction step for $\neg$. As for $\forall$, the derived rule

$$\frac{L_4 \vdash \varphi_0 \equiv \psi_0}{L_4 \vdash \forall x \varphi_0 \equiv \forall x \psi_0}$$

will do the trick. Similarly for $\exists$ and location variables.

The restriction in the theorem is necessary; for instance, $L_4 \vdash \sim\sim p \equiv p$, while $L_4 \nvdash \neg \sim\sim p \equiv \neg p$. However, using the *strong equivalence* relation of the definition below, we could have obtained an unrestricted equivalence theorem.

Definition. $\varphi \rightleftharpoons \psi$, "$\varphi$ is *strongly equivalent* to ψ", is an abbreviation of $(\varphi \equiv \psi) \wedge (\neg\varphi \equiv \neg\psi)$. In contrast, we shall call the equivalence relation $\varphi \equiv \psi$ *positive equivalence.*

In fact the next result holds also for the strong equivalence $\rightleftharpoons$, but at present we are concerned only with this weaker version:

Theorem ($\neg$ Normal Form). *For any formula φ there is a formula ψ such that $L_4 \vdash \varphi \equiv \psi$ and $\neg$ occurs only with atomic $R(\vec{\alpha})$ (located or unlocated) in ψ.*

Proof: by induction on the complexity of formulas.

Basis: Atomic formulas already satisfy the condition on ψ.

Induction: The induction step is divided into two cases.
Case 1: φ is $\neg\chi$.
We divide this into subcases depending on the principal connective or operator in χ.

(i) φ is $\neg R(\vec{\alpha})$; then φ already satisfies the condition.
(ii) φ is $\neg t$ or $\neg f$; we use the equivalences $L_4 \vdash \neg t \equiv f$ and $L_4 \vdash \neg f \equiv t$

(iii) φ is $\neg(\beta_1 \prec \beta_2)$; axiom 22, stating that $L_4 \vdash \neg(\beta_1 \prec \beta_2) \equiv\sim (\beta_1 \prec \beta_2)$, gives the result.

(iv) φ is $\neg \sim \chi_0$; we use the fact that $L_4 \vdash \neg \sim \chi_0 \equiv \chi_0$ and the induction hypothesis applied to χ_0.

(v) φ is $\neg\neg\chi_0$; use $L_4 \vdash \neg\neg\chi_0 \equiv \chi_0$ and the induction hypothesis for χ_0.

(vi) φ is $\neg(\chi_0 \wedge \chi_1)$; use $L_4 \vdash \neg(\chi_0 \wedge \chi_1) \equiv (\neg\chi_0 \vee \neg\chi_1)$ and the induction hypothesis applied to $\neg\chi_0$ and $\neg\chi_1$, i.e. $L_4 \vdash \neg\chi_0 \equiv \psi_0$ and $L_4 \vdash \neg\chi_1 \equiv \psi_1$, for suitable
ψ_0 , ψ_1, which gives $L_4 \vdash \varphi \equiv (\psi_0 \vee \psi_1)$ by the positive equivalence theorem.

(vii) φ is $\neg(\chi_0 \vee \chi_1)$; similar to (vi).

(viii) φ is $\neg\forall\alpha\chi_0$; We have $L_4 \vdash \neg\forall\alpha\chi_0 \equiv \exists\alpha\neg\chi_0$ The induction hypothesis gives $L_4 \vdash \neg\chi_0 \equiv \psi_0$ for suitable ψ_0 — the positive equivalence theorem then entails $L_4 \vdash \exists\alpha\neg\chi_0 \equiv \exists\alpha\psi_0$

(ix) φ is $\neg\exists\alpha\chi_0$; similar to (viii).

Case 2: If φ is $\sim \varphi_0$, $(\varphi_0 \wedge \varphi_1)$, $(\varphi_0 \vee \varphi_1)$, $\forall\alpha\varphi_0$, $\exists\alpha\varphi_0$, use the induction hypothesis and the positive equivalence theorem.

Let L_2 be ordinary two-valued, two-sorted first order logic. We borrow the following translation procedure from Feferman (1984).

Definition. Let φ be a formula in $\neg$ normal form. The L_2 - translation φ^* of φ is obtained by substituting a new predicate symbol P^- for occurrences $\neg P$ and P^+ for unnegated occurrences of P.

φ^* does not contain the negation symbol $\neg$ and is therefore a formula in L_2 (with $\sim$ as negation symbol). We have the following reduction theorem.

Theorem (L_4 / L_2 Reduction Theorem). *Let all formulas in Γ be in $\neg$ normal form and let $\Gamma^* = \{\varphi^* | \varphi \in \Gamma\}$. Then Γ has an L_4 model iff Γ^* has an L_2 model satisfying A23 and A24.*

Proof: The standard format of an L_2 model is a sequence $< D, L, [\![\,]\!], g >$ where $[\![\,]\!]$ interprets the relation symbols of the language (in our case $\prec$ and the pairs P^+, P^-) as sets of tuples of individuals and locations; the tuples being those sequences of locations/individuals which stand in the intended relation.

To facilitate the comparison with L_4 models we can also represent these L_2 models as structures $< D, \Lambda, R, in_2, \zeta >$ where Λ is the pair $< L, precede >$, $precede=[\prec]$, R is the set of relation symbols, either of the form P^+ or P^-, ζ is the interpretation g on individual and location constants extended by the identity map on R, and finally $in_2 = \{< (\lambda), Q, \vec{\alpha} > \mid < (\lambda), \vec{\alpha} > \in [\![Q]\!]\}$, where Q is either of the form P^+ or of the form P^-.

The basic difference between the L_2 and the L_4 language is that the former does not contain the symbol $\neg$ and has two relation symbols P^+ and P^- for each P in the L_4 language. Let $M = < D, \Lambda, R, in_2, \zeta >$ be an L_2 model satisfying $A23$ and $A24$. We then let $M' = < D, \Lambda, R', in_4, \zeta' >$ be the L_4 model where R' is the set of relation symbols P in the L_4 language; and ζ' agrees with ζ on individual and location constants, with the identity map on R'. Furthermore,

$$< (\lambda), \zeta'(P), \vec{\alpha}, 1 > \in in_4 \quad \text{iff} \quad < (\lambda), \zeta(P^+), \vec{\alpha} > \in in_2$$

$$< (\lambda), \zeta'(P), \vec{\alpha}, 0 > \in in_4 \quad \text{iff} \quad < (\lambda), \zeta(P^-), \vec{\alpha} > \in in_2$$

Since the rules for $\sim, \wedge, \vee, \forall, \exists$ are the same in L_4 as in L_2, we see that φ is true in M' iff φ^* is true in M. In a similar way we can pass from an L_4 model to an L_2 model satisfying $A23$ and $A24$. This proves the Reduction Theorem.

We conclude this section with a simple lemma.

Lemma. *If* $L_2 \vdash \varphi^*$, *then* $L_4 \vdash \varphi$.

Proof: Since L_2 is axiomatized by axiom schemas and rules already present in L_4 we obtain a proof of φ in L_4 from a proof of φ^* in L_2 by substituting $\neg P$ for P^- and P for P^+.

3.2.4 Completeness. The following *completeness result* is stated as a theorem about consistent sets of formulas. The reader should not be confused by the two uses of "consistent", corresponding to the two types of negation. In an *inconsistent situation* we can have $(\varphi \wedge \neg\varphi)$, but we *never* allow $(\varphi \wedge \sim \varphi)$. Whereas *not* $(\varphi \wedge \neg\varphi)$ follows from the constraint

$$<\vec{\alpha}, 1 > \in in \text{ implies } <\vec{\alpha}, 0 > \notin in,$$

not $(\varphi \wedge \sim \varphi)$ follows directly from the inductive rules for interpreting formulas.

Definition. A set of formulas is *(deductively) consistent* if there is no finite subset $\{\varphi_1, \ldots, \varphi_n\}$ such that $\vdash\sim (\varphi_1 \wedge \ldots \wedge \varphi_n)$. (In particular, a consistent set cannot contain both φ and $\sim \varphi$.)

Theorem. *Every set of formulas consistent in L_4 has a generalized model.*

Proof: Let Γ be consistent in L_4. We can suppose that Γ contains the universal closures of A23 and A24. By the $\neg$ normal form theorem we can suppose that all formulas of Γ are in $\neg$ normal form. Let Γ^* be the L_2-translation of Γ; by the lemma in the previous section Γ^* is consistent in L_2. Hence Γ^* has an L_2 model. This model satisfies A23 and A24. By the L_4/L_2 reduction theorem Γ has a generalized model.

Theorem. *Every set of formulas consistent in L_3 has a model.*

Proof: Let Γ be consistent in L_3. For every relation symbol P occurring in Γ we add the formula

$$(\forall \ell)\forall x_1 \ldots \forall x_n(\neg P((\ell), x_1, \ldots, x_n) \supset\sim P((\ell), x_1, \ldots, x_n)).$$

By A17b this new set is also consistent. Since it is consistent in L_3 it is also consistent in L_4. By the previous theorem it has a generalized model. Because of the added instances of A17b this model satisfies

$$<\vec{\alpha}, 0>\in in \text{ implies } <\vec{\alpha}, 1>\notin in,$$

i.e. the model is proper.

Completeness has its usual consequences, e.g. the existence of countable models. Rather than spelling these out we turn to a study of *persistence* in the present framework.

3.2.5 Persistence. The notion of persistence was discussed in section IV.2.1.; we recall the definitions: A formula is *1-persistent* iff for any two models $M =< D, R, \Lambda, in, \zeta >$ and $M' =< D, R, \Lambda, in', \zeta >$ with the same D,R,Λ,ζ and with $in \subseteq in'$, if φ is true in M with variable assignment A, then φ is true in M' with the same variable assignment. A formula φ is *0-persistent* iff $\neg\varphi$ is 1-persistent. φ is *persistent* iff it is both 0- and 1-persistent. (This definition is the same both for proper and generalized models.)

Notice that the definitions do not involve *precede*; since there is no partiality about the location structures, no new information can be added. A formula is *pure* if it contains no occurrences of the negation symbol $\sim$:

Theorem. *Every pure formula is persistent.*

Proof: This proof is by a simple induction.

We shall prove the converse theorem, that is, we prove that every persistent formula is strongly equivalent to a pure formula. A partial result in this direction follows directly from results about a similar notion in classical logic:

Roughly, φ is an *increasing formula* in a given set Q of relation symbols iff (in classical models) it remains true when more n-tuples are included in the interpretations of the relation symbols in Q. To make this more precise; if $M =< D, L, [\![\ ]\!], g >$ and $M' =< D, L, [\![\ ]\!]', g >$ are two L_2 models of the standard format, differing from each other only with respect to the interpretation of relation symbols, and $[\![P]\!] \subseteq [\![P]\!]'$ for $P \in$ Q, while $[\![P]\!] = [\![P]\!]'$ for $P \notin$ Q , then $[\![\varphi]\!]_{M,A}$ implies $[\![\varphi]\!]_{M',A}$. From the argument used to prove the L_4/L_2 reduction theorem, we see the following:

Lemma. *Let φ be a formula in $\neg$ normal form. φ is 1-persistent with respect to generalized models iff φ^* is increasing in all relation symbols but $\prec$.*

With this correspondence established, the following result becomes relevant:

Theorem. *A formula φ (in an L_2 language; hence containing only one type of negation symbol $\sim$) is increasing in Q iff there exists a formula ψ positive in Q (i.e. not containing members of Q inside the scope of any occurrence of $\sim$) such that $L_2 \vdash \varphi \equiv \psi$.*

A one-sorted version of this was proved by Roger Lyndon (1959b), as a corollary to the Craig-Lyndon interpolation theorem for first order logic. Although the argument to derive the corollary from the interpolation theorem does not assume a one-sorted language, the particular interpolation theorem in question has previously been proved only for one-sorted languages. In Appendix B we show how the many-sorted version can be derived as a corollary to the one-sorted version. From this, the corresponding characterization theorem follows by Lyndon's proof; we shall sketch his argument. We give only a simplified version, though, since our version of the characterization theorem for increasing formulas is a special case of Lyndon's more generally stated version:

Suppose φ is increasing in Q. Then $L_2 \vdash \varphi \supset (I \supset \varphi')$ where I is the conjunction of the sentences

$$\forall v_1 \ldots \forall v_n (S(v_1, \ldots, v_n) \supset S'(v_1, \ldots, v_n))$$

for $S \in Q$. Each v_i is of the appropriate sort, and S' is a new relation symbol, unique for each S. φ' is φ with S replaced by S' for all $S \in Q$. By the interpolation theorem (theorem 6.1 of Appendix B), there is a formula ψ such that

$$L_2 \vdash \varphi \supset \psi \quad \text{and} \quad L_2 \vdash \psi \supset (I \supset \varphi')$$

where

$$Rel^-(\psi) \subseteq Rel^-(\varphi) \cap Rel^-(I \supset \varphi').$$

From this it follows that

$$Rel^-(\psi) \cap (Q \cup Q') \text{ is empty.}$$

$Rel^-(\psi)$ is the set of relation symbols with occurrences in ψ inside the scope of an uneven number of occurrences of $\sim$. Hence we can assume that ψ is positive in $Q \cup Q'$. Replacing the symbols in Q' by their counterparts in Q, ψ becomes ψ_0, φ is unchanged, φ' becomes φ, I becomes a theorem, and we obtain

$$L_2 \vdash \varphi \supset \psi_0 \quad \text{and} \quad L_2 \vdash \psi_0 \supset \varphi,$$

i.e.

$$L_2 \vdash \varphi \equiv \psi_0.$$

Moreover, since ψ is positive in $Q \cup Q'$, ψ_0 is positive in Q. This completes the argument.

Combining these results, we obtain the following:

Theorem. *A formula φ is 1-persistent with respect to generalized models iff there exists a pure formula ψ such that $L_4 \vdash \varphi \equiv \psi$.*

Proof: One direction is routine. Now suppose φ is 1-persistent. φ will be positively equivalent to a formula ζ in $\neg$ normal form. Since they are positively equivalent, also ζ is 1-persistent. Now ζ^* is increasing in all relation symbols but $\prec$; hence there exists ξ such that $L_2 \vdash \zeta^* \equiv \xi$ where at most $\prec$ occurs within the scope of $\sim$ in ξ . ξ is ψ^* for a $\neg$ normal formula ψ where at most $\prec$ occurs within the scope of any $\sim$.

Now $(\zeta^* \equiv \xi)$ is $(\zeta \equiv \psi)^*$; hence $L_2 \vdash (\zeta \equiv \psi)^*$ and $L_4 \vdash \zeta \equiv \psi$. Since also $L_4 \vdash \varphi \equiv \zeta$, $L_4 \vdash \varphi \equiv \psi$. Since ψ is $\neg$ normal and at most $\prec$ occurs inside the scope of any occurrence of $\sim$ in ψ, using $A22$ and the positive equivalence theorem it is easy to see that also $L_4 \vdash \varphi \equiv \psi_0$ for a *pure* formula ψ_0.

However, we are primarily interested in formulas 1-persistent with respect to proper models. We shall obtain a similar theorem for such formulas, but first we need a definition and a lemma.

Definition. For any formula φ let $contr_\varphi$ be the disjunction of the formulas

$$(\exists \ell)\exists x_1..\exists x_n(R((\ell,)x_1,..,x_n) \wedge \neg R((\ell,)x_1,..,x_n))$$

for relation symbols R occurring in φ .

Lemma. *If φ is 1-persistent, then $\varphi \vee contr_\varphi$ is 1-persistent with respect to generalized models.*

Proof: Suppose φ is 1-persistent, M and M' are generalized models, $[\![\varphi \vee contr_\varphi]\!]^+_{M,A}$ and $in_M \subseteq in_{M'}$; we must show that $[\![\varphi \vee contr_\varphi]\!]^+_{M',A}$

(i) If $[\![contr_\varphi]\!]^+_{M',A}$, then $[\![\varphi \vee contr_\varphi]\!]^+_{M',A}$.

(ii) If *not* $[\![contr_\varphi]\!]^+_{M',A}$, we can assume both M and M' are proper. We have *not* $[\![contr_\varphi]\!]^+_{M,A}$, hence $[\![\varphi]\!]^+_{M,A}$. By 1-persistence of φ with respect to proper models, $[\![\varphi]\!]^+_{M',A}$ and $[\![\varphi \vee contr_\varphi]\!]^+_{M',A}$.

Theorem. *A formula φ is 1-persistent (with respect to proper models) iff there exists a pure formula ψ such that $L_3 \vdash \varphi \equiv \psi$.*

Proof: If φ is 1-persistent, $\varphi \vee contr_\varphi$ is 1-persistent with respect to generalized models. Therefore, there exists a pure formula ψ such that $L_4 \vdash (\varphi \vee contr_\varphi) \equiv \psi$. Since $L_3 \vdash (\varphi \vee contr_\varphi) \equiv \varphi$, we get the desired conclusion.

Corollary. *A formula φ is 0-persistent (with respect to proper models) iff there exists a pure formula ψ such that $L_3 \vdash \neg\varphi \equiv \neg\psi$.*

Combining the separate 1- and 0-persistence characterizations, we get the following characterization of full persistence:

A formula φ is persistent iff there exist pure formulas ψ and ξ such that $L_3 \vdash \varphi \equiv \psi$ and $L_3 \vdash \neg\varphi \equiv \neg\xi$.

But we can state a stronger version. The existence of *two* such formulas ψ and ξ is equivalent to the existence of a single one filling the role of both. This follows from the *relative saturation lemma* below.

Definition. A formula ψ is saturated relative to a formula φ if $L_3 \vdash \varphi \supset\!\sim \neg\psi$ implies $L_3 \vdash \varphi \supset \psi$.

Lemma. *For every two pure formulas ψ and φ , there exists a pure formula ψ' which is saturated relative to φ, such that $L_3 \vdash \neg\psi \equiv \neg\psi'$.*

For a proof, see Appendix C. We use this to prove the next result:

Lemma. *Let ψ and ξ be pure formulas. Then if $L_3 \vdash \varphi \equiv \psi$ and $L_3 \vdash \neg\varphi \equiv \neg\xi$, there exists a pure formula ζ such that $L_3 \vdash \varphi \rightleftharpoons \zeta$.*

Proof: Suppose $L_3 \vdash \varphi \equiv \psi$ and $L_3 \vdash \neg\varphi \equiv \neg\xi$ where ψ and ξ are pure formulas. By the relative saturation lemma, we can suppose that ξ is saturated relative to ψ.

We shall *almost* prove $L_3 \vdash \xi \equiv \psi$, i.e. we shall prove $L_3 \vdash \psi \supset \xi$ and $L_3 \vdash \xi \supset\!\sim \neg\psi$. Given this asymmetrical result, we can always find a pure formula, strongly equivalent to φ:

Claim 1. $L_3 \vdash \psi \supset \xi$

Proof: Since $L_3 \vdash \psi \supset \varphi$, $L_3 \vdash \varphi \supset\!\sim \neg\varphi$ and $L_3 \vdash\sim \neg\varphi \supset\!\sim \neg\xi$, we conclude that $L_3 \vdash \psi \supset\!\sim \neg\xi$. Since ξ is saturated relative to ψ , it follows that $L_3 \vdash \psi \supset \xi$.

Claim 2. $L_3 \vdash \xi \supset\!\sim \neg\psi$

Proof: Suppose not. Then $[\![\xi]\!]^+_{M,A}$ and $[\![\psi]\!]^-_{M,A}$ for some M and A. By persistence of the pure formulas ψ and ξ , we can suppose M is complete. Since $[\![\psi]\!]^-_{M,A}$, *not* $[\![\psi]\!]^+_{M,A}$. But since $L_3 \vdash \varphi \equiv \psi$, this implies *not* $[\![\varphi]\!]^+_{M,A}$. Analogously we also get *not* $[\![\varphi]\!]^-_{M,A}$. But this is impossible, since for complete models M either $[\![\varphi]\!]^+_{M,A}$ or $[\![\varphi]\!]^-_{M,A}$.

A straightforward check, e.g. by the use of truth-tables, will now verify that $L_3 \vdash \varphi \rightleftharpoons (\xi \wedge (\psi \vee \neg\psi))$.

The full persistence characterization theorem now immediately follows:

Theorem. *φ is persistent iff there exists a pure formula ζ such that $L_3 \vdash \varphi \rightleftharpoons \zeta$.*

3.3 Modal operators

We now turn to the technical study of languages with modal operators; for a general discussion see section 3.2.1. We will return to the full model structure

$$M =< S, D, \Lambda, R, In >$$

and assume that the structures satisfy the *consistency* and *compatibility constraints* discussed in section IV.3.1. above.

3.3.1 The system EL_3. In the extended L_3 language we add the modal operator $\Box$ (recalling from IV.2.1. that $\varphi \Rightarrow \psi$ can be defined as $\Box(\varphi \supset \psi)$). The formulas of EL_3 are defined as follows:

- If φ is an L_3 formula, then $\Box\varphi$ is a formula of the extended language.
- If φ and ψ are formulas, then so are $(\varphi \wedge \psi)$, $\sim \varphi$, $\forall x\varphi$, $\forall \ell\varphi$.

Observe that any occurrence of an atomic formula will be contained within the scope of exactly one occurrence of the $\Box$-operator.

For simplicity we do not introduce $\vee$ or $\exists$ as primitive symbols when occurring outside a modal operator. It is convenient, however, to allow $(\varphi \vee \psi)$, $\exists x\varphi$, $\exists \ell\varphi$ as abbreviations of $\sim (\sim \varphi \wedge \sim \psi)$, $\sim \forall x \sim \varphi$, $\sim \forall \ell \sim \varphi$. We introduce $(\varphi \supset \psi)$ and $\diamond\varphi$ as abbreviations of $(\sim \varphi \vee \psi)$ and $\sim\Box\sim \varphi$, respectively.

We give the following *satisfaction condition* for $\Box\varphi$ in a model $< S, D, \Lambda, R, In, \zeta >$ under a variable assignment A:

- $[\![\Box \varphi]\!]^+_{M,A}$ iff $[\![\varphi]\!]^+_{M_s,A}$ for all $s \in S$, where M_s is the L_3-model $< D, \Lambda, R, in_s, \zeta >$.
- $[\![\Box \varphi]\!]^-_{M,A}$ iff *not* $[\![\Box \varphi]\!]^+_{M,A}$.

The extended language has a classical logic and respects the law of the excluded middle. The interpretation rules for connectives and operators occurring outside the scope of any $\Box$ are those we know from classical logic. From the definition of $\diamond\varphi$, it will be seen that $\diamond\varphi$ is true iff φ is true in some situation. A formula is *valid* iff it is satisfied in every model under every variable assignment. The following is an axiomatization of the set of valid formulas:

Axiom schemas for EL_3:

(1) $\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$

(2) $\diamond\neg\varphi \supset \Box\sim \varphi$ $\qquad\qquad$ φ atomic

(3) $\diamond t$

(4) $\forall\alpha\varphi \supset \varphi(\beta/\alpha)$ With the same restrictions as for A25.
(5) $\forall\alpha\Box\varphi \supset \Box\forall\alpha\varphi$

Rules:

(6) If $L_3 \vdash \varphi$, then $\Box\varphi$ is a theorem.
(7) If φ follows tautologically from ψ, we can derive φ from ψ.
(8) For α not free in χ:

$$\frac{\chi \supset \varphi}{\chi \supset \forall\alpha\varphi}$$

Theorem. *The axioms are all valid, the rules preserve validity.*

The proof is straightforward. Note that (2) says that the situations are mutually consistent, (3) that there is at least one situation. Concerning (4) it should be noted that in many modal systems it is only valid with restrictions since β could be a constant with different denotations in different possible worlds. In the present system constants denote uniformly over situations; hence no such problems arise.
As a preparation for the completeness theorem we state a technical lemma.

Lemma. *The following formulas are all theorems of EL_3.*

$\Diamond(\varphi \vee \sim \varphi)$

$\Box(\varphi \supset \psi) \supset (\Diamond\varphi \supset \Diamond\psi)$

$\Box(\varphi \supset \psi) \supset (\Diamond(\chi \wedge \varphi) \supset \Diamond(\chi \wedge \psi))$

$\Box\varphi \supset (\Diamond\psi \supset \Diamond(\varphi \wedge \psi))$

$\Diamond\varphi \supset \Diamond(\varphi \wedge (\psi \vee \sim \psi))$

$\Diamond(\varphi \vee \psi) \supset (\Diamond\varphi \vee \Diamond\psi)$

$\Box(\exists\alpha\varphi \supset \exists\beta\varphi(\beta/\alpha))$ β does not occur in φ.

$\Diamond(\chi \wedge \exists\alpha\varphi) \supset \Diamond(\chi \wedge \exists\beta\varphi(\beta/\alpha))$ β does not occur in φ.

$\Box((\varphi \wedge \exists\alpha\psi) \supset \exists\alpha(\varphi \wedge \psi))$ α not free in φ.

$\Diamond(\varphi \wedge \exists\alpha\psi) \supset \Diamond\exists\alpha(\varphi \wedge \psi)$ α not free in φ.

$\Diamond\exists\alpha\varphi \supset \exists\alpha \Diamond \varphi$

The proof of the lemma is left to the reader.

3.3.2 A Completeness Theorem.

Theorem. *Every consistent set of formulas in EL_3 has a model.*

Proof: Let Γ be consistent. By a standard Henkin argument we can suppose Γ is maximally consistent and contains a witnessing formula $\exists\alpha\varphi \supset \varphi(\beta/\alpha)$ for every formula $\exists\alpha\varphi$ in the language.

For a given such Γ, we define a set Π of formulas in the language of L_3 to be a $\diamond$ *-set with respect to* Γ if for every finite subset $\{\varphi_1, ..., \varphi_n\} \subseteq \Pi$, $\diamond(\varphi_1 \wedge \ldots \wedge \varphi_n) \in \Gamma$.
If Π is contained in no other $\diamond$-set, it is called a maximal $\diamond$-set.

Claim. Every set $\{\psi\}$, with $\diamond\psi \in \Gamma$, can be extended (*without adding new variables or constants*) to a $\diamond$-set containing one formula $\exists\alpha\varphi \supset \varphi(\beta/\alpha)$ for every $\exists\alpha\varphi$-formula in the L_3 language.

Proof of the claim. Let $< \gamma_n >_{n\geq 1}$ be an enumeration of all formulas $\exists\alpha\varphi$ in L_3; we define a sequence of finite $\diamond$-sets $\{\Pi_n\}_{n\geq 0}$ each containing ψ and each containing one new Henkin-formula relative to the sequence $< \gamma_n >_{n\geq 1}$: We set $\Pi_0 = \{\psi\}$. Assume that Π_n is a finite $\diamond$-set containing ψ and $\exists\alpha\varphi\supset\varphi(\beta/\alpha)$ for the first n $\exists\alpha\varphi$ in the sequence $< \gamma_n >$. Let γ_{n+1} be $\exists\alpha\varphi$; we show how to extend Π_n to a finite $\diamond$-set containing $\gamma_{n+1} \supset\varphi(\beta/\alpha)$ for some β.
In the following we write $\bigwedge\Pi_n$ for the conjunction of the formulas in Π_n. Observe,

$$EL_3 \vdash \diamond(\bigwedge\Pi_n) \supset [\diamond(\bigwedge\Pi_n \wedge \exists\alpha\varphi) \vee \diamond(\bigwedge\Pi_n \wedge \sim \exists\alpha\varphi)]$$

Since Π_n is a $\diamond$-set, $\diamond(\bigwedge\Pi_n) \in \Gamma$. By maximal consistency of Γ, either

(i) $\diamond(\bigwedge\Pi_n \wedge \sim \exists\alpha\varphi) \in \Gamma$, or

(ii) $\diamond(\bigwedge\Pi_n \wedge \exists\alpha\varphi) \in \Gamma$

We will show that both cases imply that for some β

(iii) $\diamond(\bigwedge\Pi_n \wedge (\exists\alpha\varphi \supset (\varphi(\beta/\alpha))) \in \Gamma$.

Case (i) is obvious. In case (ii) we suppose $\diamond(\bigwedge\Pi_n \wedge \exists\alpha\varphi) \in \Gamma$, we may assume that α is not free in Π_n. Since

$$EL_3 \vdash \diamond(\bigwedge\Pi_n \wedge \exists\alpha\varphi) \supset \exists\alpha \diamond (\bigwedge\Pi_n \wedge \varphi),$$

we get $\exists\alpha \diamond (\bigwedge \Pi_n \wedge \varphi) \in \Gamma$. From the initial assumption on Γ we know that

$$\exists\alpha \diamond (\bigwedge \Pi_n \wedge \varphi) \supset \diamond(\bigwedge \Pi_n \wedge \varphi(\beta/\alpha)) \in \Gamma,$$

for some suitable β. Thus $\diamond(\bigwedge \Pi_n \wedge \varphi(\beta/\alpha)) \in \Gamma$, which implies that $\diamond(\bigwedge \Pi_n \wedge (\exists\alpha\varphi \supset \varphi(\beta/\alpha))) \in \Gamma$; thus (iii) is proved in both case (i) and case (ii).
Let $\Pi_{n+1} = \Pi_n \bigcup \{\exists\alpha\varphi \supset \varphi(\beta/\alpha)\}$. This is a finite $\diamond$-set with the desired property.
Finally, $\bigcup_n \Pi_n$ is a $\diamond$-set which satisfies all the properties of the claim.

Claim. Every $\diamond$-set can be extended to a $\diamond$-set containing for every formula φ, either φ or $\sim \varphi$.

Proof of the claim. The heart of the argument is to show that if Π is a $\diamond$-set, then either
$\Pi \bigcup \{\varphi\}$ or $\Pi \bigcup \{\sim \varphi\}$ is a $\diamond$-set.
Suppose not. Then there exist $\chi_1, \ldots, \chi_n, \xi_1, \ldots, \xi_m \in \Pi$ $(n, m \geq 0)$ such that

$$\diamond(\chi_1 \wedge \ldots \wedge \chi_n \wedge \varphi) \notin \Gamma$$

$$\diamond(\xi_1 \wedge \ldots \wedge \xi_m \wedge \sim \varphi) \notin \Gamma$$

Hence

$$\diamond(\chi_1 \wedge \ldots \wedge \chi_n \wedge \xi_1 \wedge \ldots \wedge \xi_m \wedge (\varphi \vee \sim \varphi)) \notin \Gamma$$

If $n = m = 0$, this means $\diamond(\varphi \vee \sim \varphi) \notin \Gamma$, which is impossible, since $\diamond(\varphi \vee \sim \varphi)$ is a theorem and Γ is maximal.
If $n > 0$ or $m > 0$, $\diamond(\chi_1 \wedge \ldots \wedge \chi_n \wedge \xi_1 \wedge \ldots \wedge \xi_m) \notin \Gamma$, contradicting the assumption that Π is a $\diamond$-set.

Claim. Every $\diamond$-set is consistent in L_3.

Proof of the claim. If Π is not consistent, there exist $\varphi_1, \ldots, \varphi_m \in \Pi$ such that $L_3 \vdash\sim (\varphi_1 \wedge \ldots \wedge \varphi_n)$. By rule (6) we get $\vdash\Box\sim (\varphi_1, \ldots, \varphi_m)$, i.e. $\vdash\sim \diamond(\varphi_1, \ldots, \varphi_m)$. By consistency $\diamond(\varphi_1 \wedge \ldots \wedge \varphi_n) \notin \Gamma$, which means that Π cannot be a $\diamond$-set.
We summarize the three claims for the following:

Claim. Let $\diamond\psi \in \Gamma$; then ψ is contained in a $\diamond$-set Π_ψ which is maximally consistent in L_3 and contains one formula $\exists\alpha\varphi \supset \varphi(\beta/\alpha)$ for each $\exists\alpha\varphi$ in the L_3 language.

Sets Π_ψ are called *situation sets*, and our claim says that if $\diamond\psi \in \Gamma$, then ψ is contained in some situation set Π_ψ. The reason for the name situation set is that such sets determine single-situation models: Let Π be a situation set, we define the L_3 model

$$M_\Pi = < D, R, < L, \mathit{precede} >, \mathit{in}_\Pi, \zeta >$$

by letting D,R, and L be the sets of individual variables and constants, relation symbols, and location variables and constants respectively. ζ is the identity function on constants and relation symbols. We further stipulate

$$< P, \vec{\alpha}, 1 > \in \mathit{in}_\Pi \quad \text{iff} \quad P(\vec{\alpha}) \in \Pi$$

$$< P, \vec{\alpha}, 0 > \in \mathit{in}_\Pi \quad \text{iff} \quad \neg P(\vec{\alpha}) \in \Pi$$

Since $L_3 \vdash \neg P(\alpha) \supset\sim P(\alpha)$, and Π is maximally consistent in L_3,

$$< P, \vec{\alpha}, 0 > \in \mathit{in}_\Pi \text{ implies } < P, \vec{\alpha}, 1 > \notin \mathit{in}_\Pi$$

Similarly, let β_0 *precede* β_1 iff $(\beta_0 \prec \beta_1) \in \Pi$.
By A23 and A24 $< L, \mathit{precede} >$ is a proper location structure.

From this construction it follows that $P(\vec{\alpha})$ will be true iff $P(\vec{\alpha}) \in \Pi$, $P(\vec{\alpha})$ will be false iff $\neg P(\vec{\alpha}) \in \Pi$. $(\beta_0 \prec \beta_1)$ will be true iff $(\beta_0 \prec \beta_1) \in \Pi$.
By the semantical rules, $(\beta_0 \prec \beta_1)$ is false iff $(\beta_0 \prec \beta_1)$ is not true, which holds iff $(\beta_0 \prec \beta_1) \notin \Pi$. Since $L_3 \vdash \neg(\beta_0 \prec \beta_1) \equiv \sim (\beta_0 \prec \beta_1)$ and Π is maximally consistent, $(\beta_0 \prec \beta_1) \notin \Pi$ iff $\neg(\beta_0 \prec \beta_1) \in \Pi$. Hence $(\beta_0 \prec \beta_1)$ is false iff $\neg(\beta_0 \prec \beta_1) \in \Pi$.

A straightforward induction shows that every formula has the same property; i.e. that a formula χ is true in the model iff $\chi \in \Pi$, false iff $\neg\chi \in \Pi$.

We now arrive at the final stage. Let

$$M = < S, D, R, < L, \mathit{precede} >, \mathit{In}, \zeta >$$

be the EL_3 model obtained by letting S be the set of situation sets and

$$< \Pi, \vec{\alpha} > \in \mathit{In} \quad \text{iff} \quad < \vec{\alpha} > \in \mathit{in}_\Pi$$

This is possible since the domains are the same. We need to verify the consistency constraint and that the various situation sets define the same *precedence* relation.

For the first we notice that if $P(\vec{\alpha}) \in \Pi_0$, i.e. $\diamond P(\vec{\alpha}) \in \Gamma$, then the axiom $\diamond\neg\varphi \supset \Box\sim\varphi$, i.e. $\diamond\varphi \supset \sim\diamond\neg\varphi$ (for atomic φ), and the consistency of Γ, implies that $\diamond\neg P(\vec{\alpha}) \notin \Gamma$; hence, $\neg P(\vec{\alpha}) \notin \Pi_1$, for any situation set Π_1; thus the consistency condition follows:

$$< \Pi_0, P, \vec{\alpha}, 1 > \in \mathit{In} \text{ implies that } < \Pi_1, P, \vec{\alpha}, 0 > \notin \mathit{In}$$

Likewise $(\beta_0 \prec \beta_1) \in \Pi_0$ implies $\neg(\beta_0 \prec \beta_1) \notin \Pi_1$ which again implies that $(\beta_0 \prec \beta_1) \in \Pi_1$, i.e. $\beta_0 \mathit{precede}_{\Pi_0} \beta_1$ implies $\beta_0 \mathit{precede}_{\Pi_1} \beta_1$; the precedence relations are the same.
Thus M is a legitimate EL_3 model. We arranged that if $\diamond\varphi \in \Gamma$, then φ is contained in some situation set, i.e. $\diamond\varphi$ is true in the model. If $\Box\chi \in \Gamma$, we know that $(\diamond\varphi \supset \diamond(\varphi \wedge \chi)) \in \Gamma$, hence χ is contained in all the situation sets, and is true in every situation. Hence $\Box\chi$ is true in the model. A straightforward induction shows that M is a model for Γ. This concludes the proof of the completeness theorem.
3.3.3 A Concluding Remark. With this we bring our logical investigations to a close. We believe that a many-sorted first order language with the partiality features derived from a situation semantics is a flexible tool not only for the study of natural languages but also in the study of other kinds of information processing systems.

4 ON THE MODEL THEORETIC INTERPRETATION OF SITUATION SCHEMATA

In this section we want to add a few remarks on how the semantic interpretation of situation schemata which we gave in Chapter III, is related to the logical systems of the present part. We do this by discussing a few, but representative examples. We could have given a general algorithm, but we feel that this would add little extra insight at the present stage. If one were to give the algorithmic details, one would probably want to use a logical language closer to the formalism of Barwise and Cooper (1981) than the one in IV.3. Given our format of fact schemata in III.2 the correspondence would then be immediate.

4.1 The basic correspondence

Let φ be the sentence "John kicked Pluto". In section 2.1 we used $SIT.\varphi$ to give an interpretation of φ relative to an utterance situation d, c and a described situation s:

$$d, c[\![SIT.\varphi]\!]s$$

if and only if there is anchor g on $SIT.\varphi.LOC$ such that

$$in\ s : at\ g(IND.1) : c(kick), c(John), c(Pluto); 1,$$

where g is an anchor on $SIT.\varphi.LOC$ if and only if

$$g(l_0) = l_d(\textit{the discourse location}),$$

$$c(\prec), g(IND.1), l_d; 1.$$

In the present context we assume that the utterance situation and the described situation, as well as the location part, are taken from a larger situational structure

$$M =< S, \Lambda, R, D, In > .$$

From this structure we get a model for the two-sorted language L_3 by fixing a situation s,

$$M_s =< \Lambda, R, D, in_s >,$$

where $in_s = In \upharpoonright s$ is the restriction of In to the fixed situation s.

Let *kick* be a located binary relation symbol of L_3, let *John* and *Pluto* be two individual constants, and l_d a location constant. Let $(SIT.\varphi)^*$ be the following formula of L_3:

$$(SIT.\varphi)^* : \quad \exists \ell(\ell \prec l_d \wedge kick(\ell, John, Pluto)).$$

A formula of L_3 cannot be interpreted in a model M unless we know how to interpret the constant symbols of the formula. In the present case let us use the discourse situation d and the speakers connection c to fix the interpretation of the constant symbols, i.e., we look upon d and c as maps satisfying

$$d(l_d) = l_d(\textit{the discourse location}),$$

$$c(John), c(Pluto) \in D,$$

$$c(kick) \in R_2^l.$$

This gives an expanded model structure $< M_s, d, c >$, obtained by adding the interpretation functions d, c to the model M_s. We now claim that

$$d, c[\![SIT.\varphi]\!]s$$

if and only if

$$< M_s, d, c > \models (SIT.\varphi)^*.$$

This is easily seen to be so, since the validity of $(SIT.\varphi)^*$ in $< M_s, d, c >$ means that there is some $\lambda \in \Lambda$ such that $< \lambda, l_d > \in$ *precede* and such that with this value assigned to ℓ, $kick(\ell,\ John,\ Pluto)$ is true in $< M_s, d, c >$, which by the definition of the relation in_s of M_s means that

$$in\ s : at\ \lambda : c(kick), c(John), c(Pluto); 1.$$

The correspondence can be extended, e.g. let φ' be the sentence "Every boy kicked Pluto". Then the L_3-translation is the formula:

$$(SIT.\varphi')^* : \quad \exists \ell(\ell \prec l_d \wedge \forall x(boy(x) \rightarrow kick(\ell, x, Pluto)))$$

Once more we will have an equivalence that $d, c[\![SIT.\varphi']\!]s$ if and only if $< M_s, d, c > \models (SIT.\varphi')^*$; see the discussion following (66) in III.1 .

However, $SIT.\varphi$ alone does not in every case determine an interpretation. To unravel problems of quantifier scope we needed to add a Q.MODE to the situation schema; see III.2. Translated into the present context this means that we will get different formulas for the different QMODEs. Let us illustrate this with the example (78) of III.2. The sentence φ is

Every policeman hunts a man who shot a candidate.

For simplicity let us keep the assumption of 2.2 that we have a "global" anchor on the LOC's of $SIT.\varphi$. Then we get the following two formulas corresponding to QMODE.1 and QMODE.3, respectively,

$$\begin{aligned}(SIT.\varphi)^*_1 : \ & \exists \ell_1 \exists \ell_2 (\ell_1 \circ l_d \wedge \ell_2 \prec l_d \wedge \forall x(policeman(x) \rightarrow \\ & \exists y(man(y) \wedge \exists z(candidate(z) \wedge shoot(\ell_2, y, z)) \wedge hunt(\ell_1, x, y)))).\end{aligned}$$

$$\begin{aligned}(SIT.\varphi)^*_3 : \ & \exists \ell_1 \exists \ell_2 (\ell_1 \circ l_d \wedge \ell_2 \prec l_d \wedge \exists z(candidate(z) \wedge \forall x(policeman(x) \rightarrow \\ & \exists y((man(y) \wedge shoot(\ell_2, y, z)) \wedge hunt(\ell_1, x, y))))).\end{aligned}$$

We leave to the readers to write down the formula $(SIT.\varphi)^*_2$ corresponding to QMODE.2 .

4.2 *The Correspondence Extended*

The reader will observe how we in the transition from M to M_s have kept the Λ, R and D of the larger structure unchanged, but have cut down the In relation of M to the relation $in_s = In \upharpoonright s$ of M_s. Thus M_s is not a model for the described situation only. We have used the d and the c of the utterance situation in the interpretation, in particular, subformulas such as $\ell_1 \circ l_d$ and $\ell_2 \prec l_d$ do not at all refer to the described situation. But every fact (located or unlocated) must be a fact of the described situation, i.e. a fact validated by the relation in_s.

This is a restriction. In the RMODE we typically want to refer to a "resource" situation in a way independent of the described situation. One could think of several ways of extending the present somewhat restricted correspondence to one within the full model structure M, where we in the presence of the set S can at the same time refer to different situations; both the utterance situation and the described situation, as well as a possible resource situation, must be members of S.

Another rather straight-forward extension of the formalism L_3 is the addition of extra generalized quantifiers. Doing this one should also explore the possibility of using a logical formalism closer to the one developed in Barwise and Cooper (1981) for the analysis of generalized quantifiers. In a sense, we did exactly this in the formalism of "complex fact schemata" developed in section III.2 for the interpretation of situation schemata.

Remark In a discussion of direct questions Vestre (1987) has used the logic L_3 in a more substantive fashion. Vestre represents direct questions as a special kind of "incomplete" situation schemata which interact with a data-base to produce answers, i.e. "completed" schemata. Constructing an algorithm which transforms situation schemata into L_3-formulas he is able to use the proof theory developed for L_3—in particular the $\neg$ Normal form—to obtain a PROLOG implementation of a natural language question answering system.

CHAPTER V

CONCLUSIONS

It is important for any semantic theory with purported relevance for natural language that it be integrated with explicit theories of linguistic form which can provide linguistically adequate analyses for a representative range of natural language phenomena. One reason for the great impact which Montague Grammar has had on linguistic semantics is the high degree to which Montague's semantic approach allowed incorporation with current syntactic analyses. But, as we stressed in the Introduction, other aspects of linguistic form in addition to syntax must be taken into account in the semantic analysis.

The fragments in II.5 and Appendix A are our examples of combining situation semantics with a syntactic analysis. We have assumed a lexical-functional grammar for the syntactic part, but it should be clear that this is not essential. Any constraint-based syntactic theory is equally amenable to this kind of treatment. As the appendix in particular shows, the semantic schemata are described by a set of equations that is separate from the functional descriptions. The information about the proper matching of relations and arguments which is crucial for interpretation could be derived from alternative syntactic, morphological and phonological analyses.

We have in this work presented an algorithm for converting linguistic form to the semantic format of situation schemata. A situation schema has a well-defined (algebraic) structure, suggestive of "logical form". This is a structure different from the standard model-theoretic one. The latter is intended as a "slice" of the real world and may carry a rich non-linguistic structure, e.g. the correct classification of verb-phrases and prepositional phrases describing spatio-temporal processes necessitates endowing the location set, Γ, with a rich geometric structure to account for the "singularities" involved. Thus model theory is an indispensable part of the enterprise. And as the completeness theorems of section

IV.3 show, our use of partiality leads to a well-understood mathematical theory.

But situation schemata can be put to other uses. They are not always "semantically complete" in the sense that they are interpretable in a model structure, M; they may e.g. leave questions of quantifier scope undetermined. We are investigating the effects of such underspecification on the use of situation schemata in inference mechanisms. We are also investigating the use of this kind of partiality in the analysis of direct questions, which do not have a conventional model theoretic semantics, but can be associated with "incomplete" situation schemata in a systematic way. These incomplete schemata suggest the range of appropriate answers (which do have model theoretic interpretations).

We have in this work restricted ourselves to basic facts of linguistic form and semantic structure. Our aim has been to demonstrate in precise technical details how to join the two. We believe that we have achieved this goal in a way which is extendable and thus open to further experimentation.

APPENDIX A

PREPOSITIONAL PHRASES IN SITUATION SCHEMATA

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In this appendix I propose an analysis of a few English sentences containing prepositional phrases (PP) leading to a situation semantic interpretation. The PP's we will consider here will all be directly connected to a verb phrase (VP) and readings where a PP combines with a noun phrase (NP) to form a more complex NP will be omitted. The PP's will be divided into two classes: oblique objects and adjuncts (see Kaplan & Bresnan 82). An oblique object fills one of the argument slots of the verb, when one considers the verb as a relation which takes a fixed number of arguments. In e.g. the sentence "Tom handed the book to Anne" the verb *handed* is a ternary relation with the arguments *Tom*, *the book* and, one might say, *Anne*. However, I will consider the third argument here to be something which is in the relation **to** to **Anne**. An oblique object is thus a constraint on an (unexpressed) argument of the verb. This way of viewing things permits a VP to have several oblique objects without increasing the number of arguments. In the sentence "Tom sent a letter from Norway to France" both *from Norway* and *to France* will be constraints on the same argument; call it the "trajectory".

Adjuncts function often by restricting or modifying the relation expressed by the verb. Examples of this is: "Tom played with Anne" and "Tom ate in a hurry". Sometimes, though, it is the location where the relation takes place that is modified, rather than the relation itself. These adverbials comprise the locative prepositional phrases. An adjunct given by a locative PP can therefore be represented in a situation schema by adding an extra member in the COND of the LOC of the schema. Consider e.g. "Tom ran to the school". The location indeterminate will here be further restricted by an extra condition saying that this indeterminate must be in the relation **to** to **the school**. This relation will hold if the location is a curve tracing the trajectory in space-time that ends at the (location of)the school.

1 WHICH PP'S ARE OBLIQUE OBJECTS AND WHICH ARE ADJUNCTS?

Usually an indirect object is considered to be an actor or argument of the verb. Therefore, when a PP can be replaced by an indirect object as in the sentences "Peter gave the book to Anne" / "Peter gave Anne the book", it will be taken as an oblique object. In other cases it can be more difficult to decide whether to consider a PP as an oblique object or as an adjunct. If one omits one of the arguments in a VP, the VP may be of doubtful grammaticality or its meaning may change in unexpected ways, cf. "Peter lives in this house" and "Peter lives." Adjuncts, on the other hand, may be omitted freely; cf. "Peter played in the garden" vs. "Peter played" and "Tom ran to school" vs. "Tom ran."

But, in certain contexts oblique objects may be omitted. Suppose e.g. that Anne has invited some friends to her birthday party. All the guests give here a birthday present. After the party somebody asks: "What did *Peter* give?". Anne answers:"He gave a book". It seems that one argument can be dropped if the focus is on one of the other arguments or if it is obvious what the missing argument should be.

In addition to the differences mentioned above there may also be some semantical differences between an oblique object-reading and an adjunct-reading. Consider e.g. the sentence:"Peter laid the book on the table". If *on the table* is read as an adjunct, the location of the described situation is constrained to be *on the table* and one would therefore think that Peter was standing, sitting or somehow placed on the table when he laid the book. This reading is unnatural. In the oblique object-reading *lay* will have three arguments, and the third one will be the location where the second argument (the book) will be situated *after* the laying has taken place. The PP *on the table* constrains this location which will have to be in the relation **on** to **the table**. In the sentence "Peter laid the book on the table on top of all the newspapers" the location where the book ended up is both **on the table** and **on top of all the newspapers**. The verb *lay* has still only three arguments.

In some cases it is difficult to observe any semantical difference between an oblique object-reading and an adjunct-reading. An example of this is : "The book was lying on the table". An adjunct-reading here would give: At the location which is on the table the book was lying. The oblique object-reading would let the verb *lie* be a binary relation between **the book** and a location which is constrained to be **on the table**. There is no obvious preferred reading this time. One may ask what the valence of *lie* is, i.e. the number of arguments the verb can have. If *lie* is a unary relation, the oblique object-reading of *on the table* must be discarded. There are some reasons, though, for considering *lie* as a binary relation. First, sentences like "The book was lying" are odd. Second, since *lie* is so closely related with *lay* in meaning, and *on the table* fills the place of an argument in "Peter laid the book on the table", one might argue that so should the PP in "The book was lying on the table". However, I shall not decide what the right thing to do is, but consider both the oblique object-reading and the adjunct-reading when the situation schemata are worked out.

Finally, consider the sentence: "Anne sent a courier from London". If *from London* is taken as an adjunct, then this means that the sending took place **from London**, i.e. Anne was in London when she sent the courier. But Anne may have been in Copenhagen, phoned to London and ordered a courier to take a message to Paris. In this case both sentences "Anne sent a courier from Copenhagen" and "Anne sent a courier from London" turn out true. In the first sentence the PP is an adjunct, in the second it is an oblique object. The verb *send* takes three arguments and the third is the "trajectory" of the second. The trajectory is here constrained to be in the relation **from** to **London**. This last sentence illustrates that both an adjunct-reading and an oblique object-reading can be plausible, even if they yield different interpretations.

2 A GRAMMAR OF A FRAGMENT OF ENGLISH CONTAINING PP'S

We now present a grammar which incorporates the above analysis of prepositional phrases. We don't give the equations for generating the f-structures since the purpose here is to focus on the situation schemata. The equations here will describe the situation schemata directly.

In the rules below the category PPOBL denotes a PP with an oblique object-reading, while PPLOC denotes a locative PP with an adjunct-reading. The category FPPOBL (= Fake PPOBL) always dominates an NP which is an indirect object of the verb. The purpose of FPPOBL is to treat the NP it dominates as if it were the NP in a PPOBL. Both PPOBL and PPLOC generate a member of a COND whose REL's value is provided by the preposition (P) in the PP. In the case of an FPPOBL (which simply is a PPOBL without a P) the REL's value is provided by the verb in the dominating VP. The transfer of this value is done through the attribute DEFAULTREL. Because this transfer cannot be done directly (for "technical" reasons) the situation schema will contain the attribute DEFAULTREL, but this attribute should be ignored when interpreting the situation schema.

Syntactic rules and lexical entries

$$\begin{array}{ll} \text{FPPOBL} \rightarrow & \text{NP} \\ & (\uparrow \text{ARG.2}) = \downarrow \end{array}$$

$$\text{NP} \rightarrow \left\{ \begin{array}{l} \text{(DET) N} \\ \text{NPROP} \end{array} \right\}$$

$$\begin{array}{ll} \text{PPLOC} \rightarrow & \text{P NP} \\ & \quad (\uparrow \text{ARG.2}) = \downarrow \end{array}$$

$$\begin{array}{ll} \text{PPOBL} \rightarrow & \text{P NP} \\ & \quad (\uparrow \text{ARG.2}) = \downarrow \end{array}$$

$$\begin{array}{lll} \text{S} \rightarrow \text{NP} & \text{(AUX)} & \text{VP} \\ \quad (\uparrow \text{ARG.1}) = \downarrow & & \uparrow = \downarrow \end{array}$$

$$\text{VP} \rightarrow \text{V} \left\{ \begin{array}{ll} \text{FPPOBL} & \text{NP} \\ \downarrow \in (\uparrow \text{ARG.3 COND}) & (\uparrow \text{ARG.2}) = \downarrow \\ (\uparrow \text{ARG.3 IND}) = (\downarrow \text{ARG.1}) & \\ (\uparrow \text{ARG.3 DEFAULTREL}) = (\downarrow \text{REL}) & \\ (\downarrow \text{POL}) = 1 & \\ \left(\begin{array}{l} \text{NP} \\ (\uparrow \text{ARG.2}) = \downarrow \end{array} \right) & \end{array} \right\}$$

$$\begin{array}{ll} \text{PPOBL*} & \text{PPLOC*} \\ \downarrow \in (\uparrow \text{ARG.3 COND}) & \downarrow \in (\uparrow \text{LOC COND}) \\ (\uparrow \text{ARG.3 IND}) = (\downarrow \text{ARG.1}) & (\uparrow \text{LOC IND}) = (\downarrow \text{ARG.1}) \end{array}$$

A	DET	*
		(↑ SPEC)=A
ANNE	NPROP	*
		(↑ IND)='ANNE'

BOOK	N	* (↑ IND)=IND1 (↑ COND REL)='BOOK' (↑ COND ARG.1)=(↑ IND) (↑ COND POL)=1
FROM	P	* (↑ REL)='FROM'
GAVE	V	* (↑ REL)='GIVE' ↓∈(↑ LOC COND) (↓ REL)=PRECEDE (↑ LOC IND)=IND2 (↓ ARG.1)=(↑ LOC IND) (↓ ARG.2)=lo (↑ POL)=1 { (↑ ARG.3 IND)=IND5 (↑ ARG.3 COND) (↑ ARG.3 DEFAULTREL)='TO' }
JOHN	NPROP	* (↑ IND)='JOHN'
KICKED	V	* (↑ REL)='KICK' ↓∈(↑ LOC COND) (↓ REL)=PRECEDE (↑ LOC IND)=IND2 (↓ ARG.1)=(↑ LOC IND) (↓ ARG.2)=lo (↑ POL)=1
LAID	V	* (↑ REL)='LAY' ↓∈(↑ LOC COND) (↓ REL)=PRECEDE (↑ LOC IND)=IND2 (↓ ARG.1)=(↑ LOC IND) (↓ ARG.2)=lo (↑ POL)=1

		$\left\{\begin{array}{l}(\uparrow \text{ ARG.3 IND})=\text{IND5} \\ (\uparrow \text{ ARG.3 COND }) \\ \ \end{array}\right\}$
LAY	V	*
		(↑ REL)='LIE'
		↓∈(↑ LOC COND)
		(↓ REL)=PRECEDE
		(↑ LOC IND)=IND2
		(↓ ARG.1)=(↑ LOC IND)
		(↓ ARG.2)=lo
		(↑ POL)=1
		$\left\{\begin{array}{l}(\uparrow \text{ ARG.3 IND})=\text{IND5} \\ (\uparrow \text{ ARG.3 COND }) \\ \ \end{array}\right\}$
LETTER	N	*
		(↑ IND)=IND3
		(↑ COND REL)='LETTER'
		(↑ COND ARG.1)=(↑ IND)
		(↑ COND POL)=1
LYING	V	*
		(↑ REL)='LIE'
		$\left\{\begin{array}{l}(\uparrow \text{ ARG.3 IND})=\text{IND5} \\ (\uparrow \text{ ARG.3 COND }) \\ \ \end{array}\right\}$
ON	P	*
		(↑ REL)='ON'
PETER	NPROP	*
		(↑ IND)='PETER'
PLUTO	NPROP	*
		(↑ IND)='PLUTO'
RAN	V	*
		(↑ REL)='RUN'
		↓∈(↑ LOC COND)
		(↓ REL)=PRECEDE
		(↑ LOC IND)=IND2
		(↓ ARG.1)=(↑ LOC IND)
		(↓ ARG.2)=lo
		(↑ POL)=1

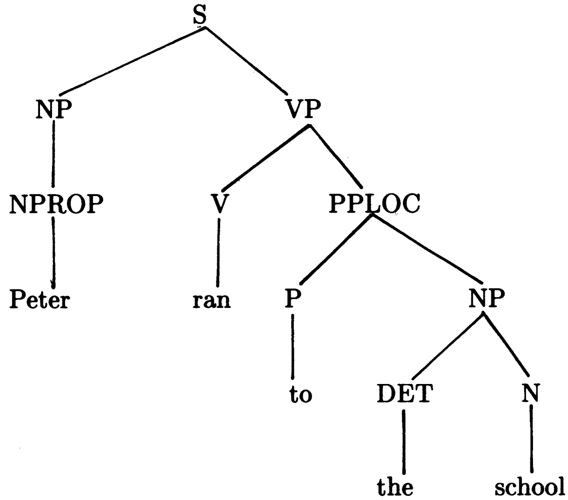
SCHOOL	N	* (↑ IND)=IND1 (↑ COND REL)='SCHOOL' (↑ COND ARG.1)=(↑ IND) (↑ COND POL)=1
SENT	V	* (↑ REL)='SEND' ↓∈(↑ LOC COND) (↓ REL)=PRECEDE (↑ LOC IND)=IND2 (↓ ARG.1)=(↑ LOC IND) (↓ ARG.2)=lo (↑ POL)=1 $\left\{\begin{array}{l}(\uparrow \text{ ARG.3 IND})=\text{IND5}\\(\uparrow \text{ ARG.3 COND })\\(\uparrow \text{ ARG.3 DEFAULTREL})=\text{'TO'}\end{array}\right\}$
STREET	N	* (↑ IND)=IND4 (↑ COND REL)='STREET' (↑ COND ARG.1)=(↑ IND) (↑ COND POL)=1
TABLE	N	* (↑ IND)=IND4 (↑ COND REL)='TABLE' (↑ COND ARG.1)=(↑ IND) (↑ COND POL)=1
THE	DET	* (↑ SPEC)=THE
TO	P	* (↑ REL)='TO'
WAS	AUX	* ↓∈(↑ LOC COND) (↓ REL)=PRECEDE (↑ LOC IND)=IND2 (↓ ARG.1)=(↑ LOC IND)

($\downarrow$ ARG.2)=lo
($\uparrow$ POL)=1

Examples of situation schemata of some sentences that contain PP:

(1) Peter ran to the school.

C-structure:



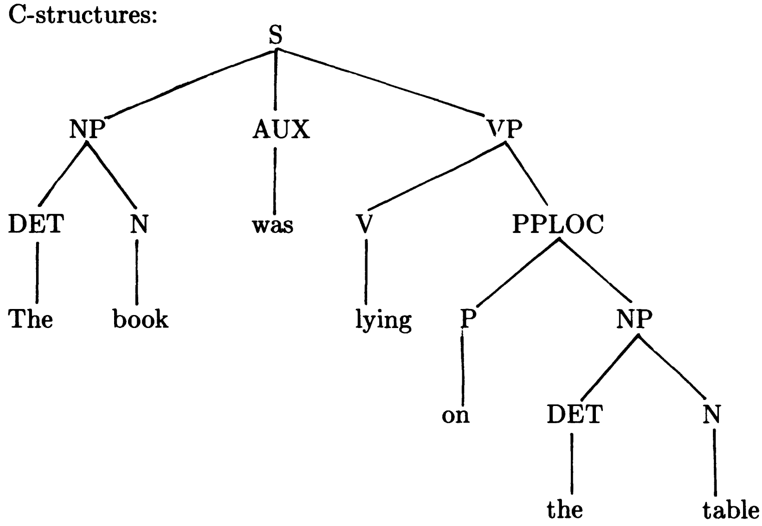
Situation Schema:

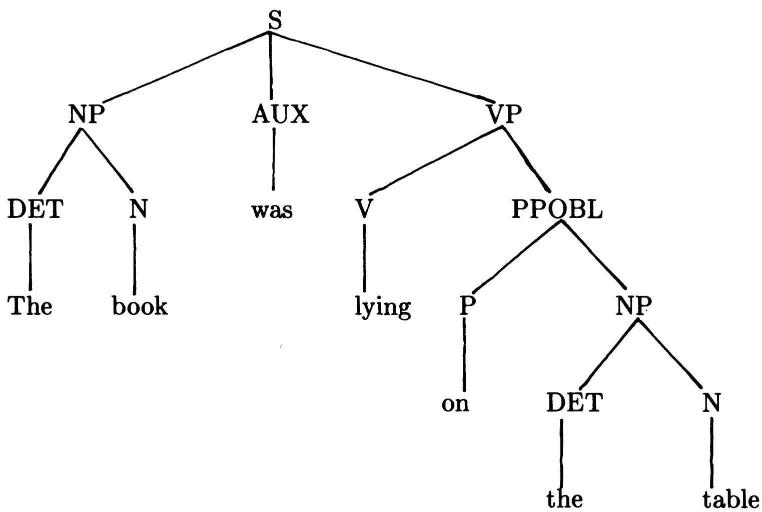
$$
\begin{bmatrix}
\text{REL} & \text{'run'} \\
\text{ARG.1} & \begin{bmatrix}\text{IND} & \text{'peter'}\end{bmatrix} \\
\text{LOC} & \begin{bmatrix}
\text{IND} & \text{IND2} \\
\text{COND} & \left\{ \begin{matrix}
\begin{bmatrix} \text{REL} & \prec \\ \text{ARG.1} & [\,]_2 \\ \text{ARG.2} & l_0 \end{bmatrix} \\
\begin{bmatrix} \text{REL} & \text{'to'} \\ \text{ARG.1} & [\,]_2 \\ \text{ARG.2} & \begin{bmatrix} \text{IND} & \text{IND1} \\ \text{COND} & \begin{bmatrix} \text{REL} & \text{'school'} \\ \text{ARG.1} & [\,]_1 \\ \text{POL} & 1 \end{bmatrix} \\ \text{SPEC} & \text{the} \end{bmatrix} \end{bmatrix}
\end{matrix} \right\}
\end{bmatrix} \\
\text{POL} & 1
\end{bmatrix}
$$

The PP is here taken as an adjunct since ran is a unary relation. The LOC in the situation schema of the sentence gets an extra member in its COND besides the one coming from the verb's tense. Since an unlimited number of adjuncts can be added to a VP, the COND in the LOC is a set (notice the curly-brackets).

(2) The book was lying on the table.

C-structures:





Situation Schema:

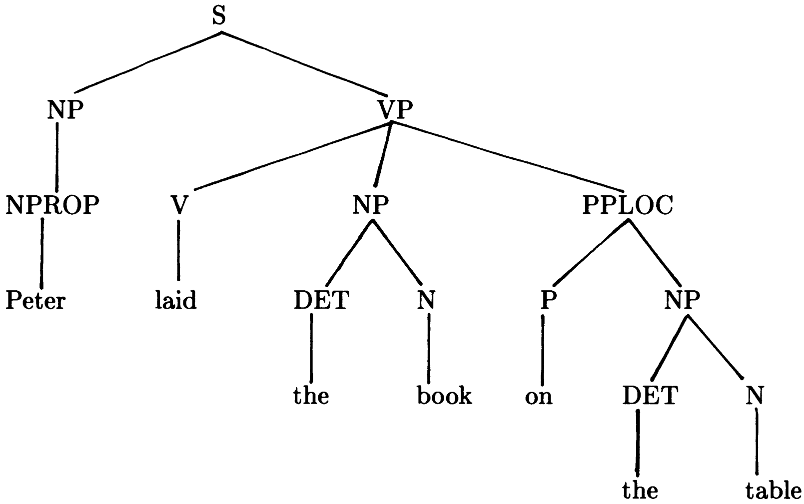
$$\begin{bmatrix}
\text{REL} & \text{`lie'} \\
\text{ARG.1} & \begin{bmatrix}
\text{IND} & \text{IND1} \\
\text{COND} & \begin{bmatrix} \text{REL} & \text{`book'} \\ \text{ARG.1} & [\,]_1 \\ \text{POL} & 1 \end{bmatrix} \\
\text{SPEC} & \text{the}
\end{bmatrix} \\
\text{LOC} & \begin{bmatrix}
\text{IND} & \text{IND2} \\
\text{COND} & \left\{ \begin{matrix}
\begin{bmatrix} \text{REL} & \prec \\ \text{ARG.1} & [\,]_2 \\ \text{ARG.2} & l_0 \end{bmatrix} \\
\begin{bmatrix}
\text{REL} & \text{`on'} \\
\text{ARG.1} & \text{IND2} \\
\text{ARG.2} & \begin{bmatrix}
\text{IND} & \text{IND4} \\
\text{COND} & \begin{bmatrix} \text{REL} & \text{`table'} \\ \text{ARG.1} & [\,]_4 \\ \text{POL} & 1 \end{bmatrix} \\
\text{SPEC} & \text{the}
\end{bmatrix}
\end{bmatrix}
\end{matrix} \right\}
\end{bmatrix} \\
\text{POL} & 1
\end{bmatrix}$$

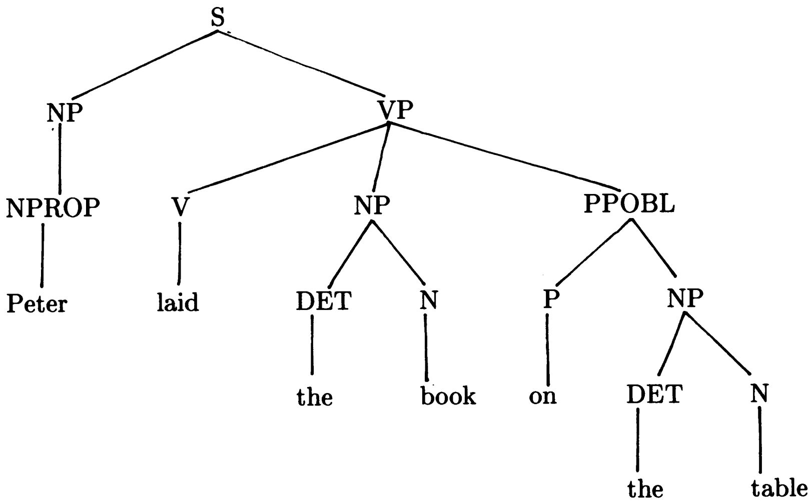
$$\begin{bmatrix}
\text{REL} & \text{'lie'} \\
\text{ARG.1} & \begin{bmatrix} \text{IND} & \text{IND1} \\ \text{COND} & \begin{bmatrix} \text{REL} & \text{'book'} \\ \text{ARG.1} & [\,]_1 \\ \text{POL} & 1 \end{bmatrix} \\ \text{SPEC} & \text{the} \end{bmatrix} \\
\text{ARG.3} & \begin{bmatrix} \text{IND} & \text{IND5} \\ \text{COND} & \begin{bmatrix} \text{REL} & \text{'on'} \\ \text{ARG.1} & [\,]_5 \\ \text{ARG.2} & \begin{bmatrix} \text{IND} & \text{IND4} \\ \text{COND} & \begin{bmatrix} \text{REL} & \text{'table'} \\ \text{ARG.1} & [\,]_4 \\ \text{POL} & 1 \end{bmatrix} \\ \text{SPEC} & \text{the} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\
\text{LOC} & \begin{bmatrix} \text{IND} & \text{IND2} \\ \text{COND} & \begin{bmatrix} \text{REL} & \prec \\ \text{ARG.1} & [\,]_2 \\ \text{ARG.2} & l_0 \end{bmatrix} \end{bmatrix} \\
\text{POL} & 1
\end{bmatrix}$$

The PP gets two readings; one as an adjunct (PPLOC) and one as an oblique object (PPOBL). The first situation schema is quite similar to the situation schema of sentence (1), but in the second there are two arguments: ARG.1 and ARG.3. One might ask whether IND5 in the second schema shouldn't be constrained further. According to the analysis done in section 1 of this appendix, ARG.3 should be the location of ARG.1, but this constraint doesn't seem to appear in the situation schema. The answer is that this constraint is contained in the situation schema in the relation **lie**. The meaning of *lie* could be given in the

following way: ARG.1 is *lying* & ARG.1's location is ARG.3. (See section A.4. Interpreting the situation schemata.)
(3) Peter laid the book on the table.

C-structures:




S
NP
VP
NPROP
V
NP
PPOBL
Peter
laid
DET
N
P
NP
the
book
on
DET
N
the
table

Situation Schemata:

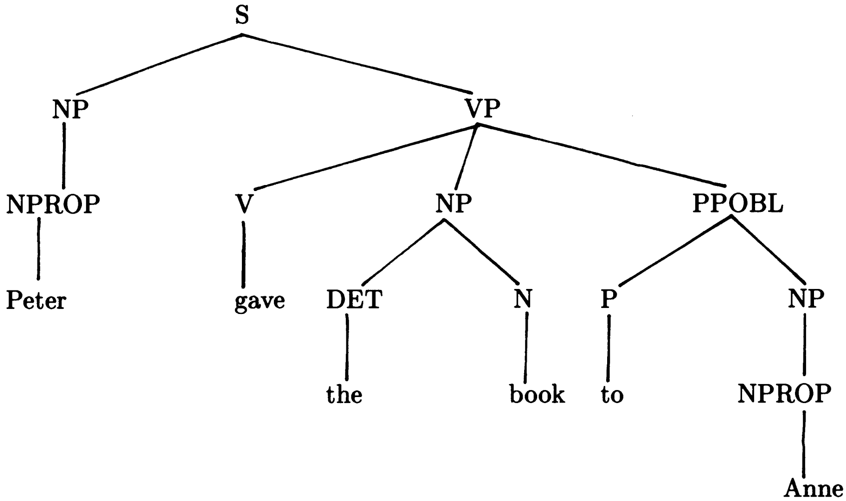
$$
\begin{bmatrix}
\text{REL} & \text{'lay'} \\
\text{ARG.1} & \begin{bmatrix} \text{IND} & \text{'Peter'} \end{bmatrix} \\
\text{ARG.2} & \begin{bmatrix}
\text{IND} & \text{IND1} \\
\text{COND} & \begin{bmatrix} \text{REL} & \text{'book'} \\ \text{ARG.1} & [\,]_1 \\ \text{POL} & 1 \end{bmatrix} \\
\text{SPEC} & \text{the}
\end{bmatrix} \\
\text{LOC} & \begin{bmatrix}
\text{IND} & \text{IND2} \\
\text{COND} & \left\{ \begin{matrix}
\begin{bmatrix} \text{REL} & \prec \\ \text{ARG.1} & [\,]_2 \\ \text{ARG.2} & l_0 \end{bmatrix} \\
\begin{bmatrix}
\text{REL} & \text{'on'} \\
\text{ARG.1} & [\,]_2 \\
\text{ARG.2} & \begin{bmatrix}
\text{IND} & \text{IND4} \\
\text{COND} & \begin{bmatrix} \text{REL} & \text{'table'} \\ \text{ARG.1} & [\,]_4 \\ \text{POL} & 1 \end{bmatrix} \\
\text{SPEC} & \text{the}
\end{bmatrix}
\end{bmatrix}
\end{matrix} \right\}
\end{bmatrix} \\
\text{POL} & 1
\end{bmatrix}
$$

$$\left[\begin{array}{ll}
\text{REL} & \text{`lay'} \\
\text{ARG.1} & \left[\begin{array}{ll}\text{IND} & \text{`Peter'}\end{array}\right] \\
\text{ARG.2} & \left[\begin{array}{ll}
\text{IND} & \text{IND1} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \text{`book'} \\
\text{ARG.1} & [\,]_1 \\
\text{POL} & 1
\end{array}\right] \\
\text{SPEC} & \text{the}
\end{array}\right] \\
\text{ARG.3} & \left[\begin{array}{ll}
\text{IND} & \text{IND5} \\
\text{COND} & \left[\begin{array}{ll}
\text{ARG.1} & [\,]_5 \\
\text{ARG.2} & \left[\begin{array}{ll}
\text{IND} & \text{IND4} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \text{`table'} \\
\text{ARG.1} & [\,]_4 \\
\text{POL} & 1
\end{array}\right] \\
\text{SPEC} & \text{the}
\end{array}\right] \\
\text{REL} & \text{`on'}
\end{array}\right]
\end{array}\right] \\
\text{LOC} & \left[\begin{array}{ll}
\text{IND} & \text{IND2} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \prec \\
\text{ARG.1} & [\,]_2 \\
\text{ARG.2} & l_0
\end{array}\right]
\end{array}\right] \\
\text{POL} & 1
\end{array}\right]$$

Except for the argument ARG.2, this sentence is quite similar to sentence (2). We get two readings again, even if the adjunct-reading is not very plausible. We may imagine the functional equations ruling out the adjunct-reading. Since the functional equations are omitted in this grammar all PP's get the adjunct-reading, but in the examples below we will not display he c-structures and schemata when this reading is unnatural.

(4) Peter gave the book to Anne.

C-structure:



Situation schema:

$$\left[\begin{array}{ll}
\text{REL} & \text{`give'} \\
\text{ARG.1} & \left[\begin{array}{ll}\text{IND} & \text{`Peter'}\end{array}\right] \\
\text{ARG.2} & \left[\begin{array}{ll}
\text{IND} & \text{IND1} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \text{`book'} \\
\text{ARG.1} & [\ ]_1 \\
\text{POL} & 1
\end{array}\right] \\
\text{SPEC} & \text{the}
\end{array}\right] \\
\text{ARG.3} & \left[\begin{array}{ll}
\text{DEFAULTREL} & \text{`to'} \\
\text{IND} & \text{IND5} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \text{`to'} \\
\text{ARG.1} & [\ ]_5 \\
\text{ARG.2} & \left[\begin{array}{ll}\text{IND} & \text{`Anne'}\end{array}\right]
\end{array}\right]
\end{array}\right] \\
\text{LOC} & \left[\begin{array}{ll}
\text{IND} & \text{IND2} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \prec \\
\text{ARG.1} & [\ ]_2 \\
\text{ARG.2} & l_0
\end{array}\right]
\end{array}\right] \\
\text{POL} & 1
\end{array}\right]$$

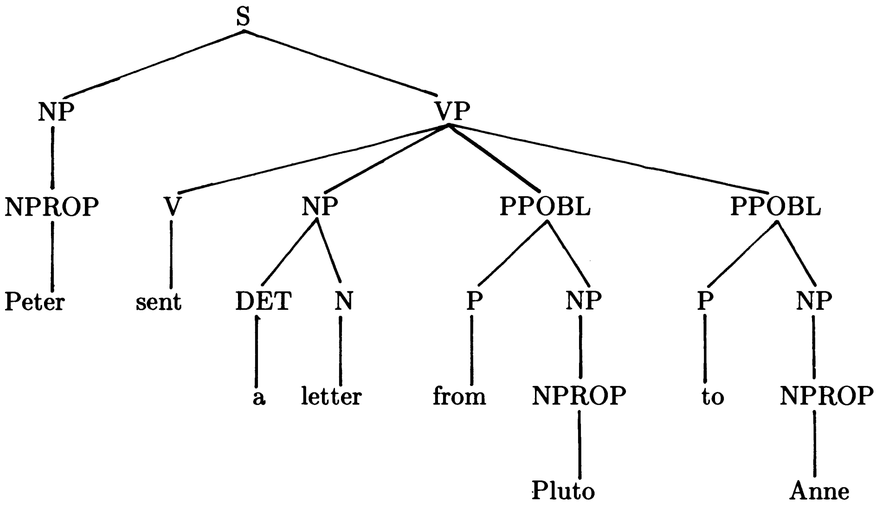
Notice the DEFAULTREL in ARG.3. This is part of a mechanism to generate the same situation schema when the third argument appears in an indirect object position. The sentence "Peter gave Anne the book" will thus have the same situation schema. The DEFAULTREL appears in the lexical entries of all verbs that can take an indirect object. In English the DEFAULTREL is always **to**, but in other languages it may vary, cf. "Der Mann hat ihm etwas gestohlen"/"Der Mann hat etwas **von** ihm gestohlen" and "Der Mann hat ihm etwas gegeben"/"Der Mann hat etwas **zu** ihm gegeben".

(5) Peter gave Anne the book.

Same situation schema as above.

(6) Peter sent a letter from Pluto to Anne.

C-structure:



Situation Schema:

$$
\left[\begin{array}{ll}
\text{REL} & \text{'send'} \\
\text{ARG.1} & \left[\begin{array}{ll} \text{IND} & \text{'Peter'} \end{array}\right] \\
\text{ARG.2} & \left[\begin{array}{ll}
\text{IND} & \text{IND3} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \text{'letter'} \\
\text{ARG.1} & [\;]_3 \\
\text{POL} & 1
\end{array}\right] \\
\text{SPEC} & \text{a}
\end{array}\right] \\
\text{ARG.3} & \left[\begin{array}{ll}
\text{DEFAULTREL} & \text{'to'} \\
\text{IND} & \text{IND5} \\
\text{COND} & \left\{\begin{array}{l}
\left[\begin{array}{ll}
\text{REL} & \text{'from'} \\
\text{ARG.1} & [\;]_5 \\
\text{ARG.2} & \left[\begin{array}{ll} \text{IND} & \text{'Pluto'} \end{array}\right]
\end{array}\right] \\
\left[\begin{array}{ll}
\text{REL} & \text{'to'} \\
\text{ARG.1} & [\;]_5 \\
\text{ARG.2} & \left[\begin{array}{ll} \text{IND} & \text{'Anne'} \end{array}\right]
\end{array}\right]
\end{array}\right\}
\end{array}\right] \\
\text{LOC} & \left[\begin{array}{ll}
\text{IND} & \text{IND2} \\
\text{COND} & \left[\begin{array}{ll}
\text{REL} & \prec \\
\text{ARG.1} & [\;]_2 \\
\text{ARG.2} & l_0
\end{array}\right]
\end{array}\right] \\
\text{POL} & 1
\end{array}\right]
$$

This example shows a COND of ARG.3 containing more than one element.

3 INTERPRETING THE SITUATION SCHEMATA

Here we will give the interpretation of a couple of sentences chosen among the examples above, thus illustrating how to work out the interpretation in general for sentences containing PP's. Both adjuncts and oblique objects will be represented. We refer to the situation schemata displayed above when interpreting these sentences.

(1) Peter ran to the school.

The meaning of this sentence is a relation $[\![SIT.1]\!]$ between an utterance situation u ($= d, c$) in $MF_{[\![SIT.1]\!]}$ and a described situation s, where the relation $u[\![SIT.1]\!]s$ holds if and only if:

there exist an anchor g on *SIT.1.LOC*, i.e.

$$g(l_0) = l_d \text{ and}$$

$$g(IND2) \prec g(l_0)$$

and an extension f of g that anchors *IND1*

such that *f(IND1)* is the unique individual

such that *in s : c(school),f(IND1);1*

such that

in s : c(to),g(IND2),f(IND1);1

in s : at g(IND2) : c(run),c(Peter);1

The $\prec$-relation in

$$g(IND2) \prec g(l_0)$$

is the "temporally precedes"-relation that holds if $g(IND2)_1$ i.e. the projection of *g(IND2)* on the time-dimension of time-space precedes $g(l_0)_1$. The relation *c(to)* in

in s : c(to),g(IND2),f(IND1);1

holds if *g(IND2)* is a curve in space-time that ends at the location of *f(IND1)*. If one wants to be more precise, one has to take into account that, for any instant in time, *f(IND1)* has an extension in space, and therefore the "curve" *g(IND2)* will have an extension, too.

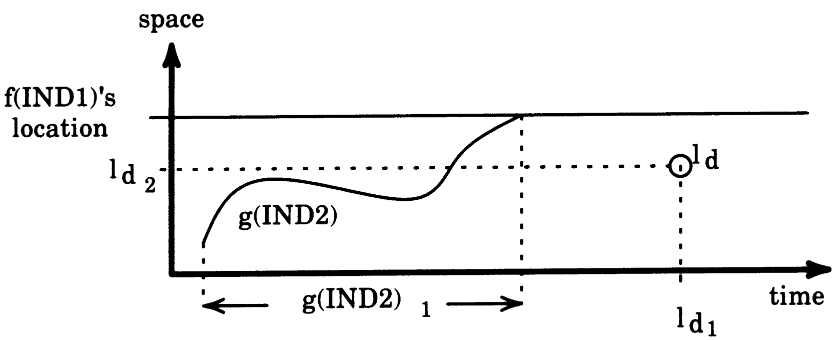
The three constraints

$$g(l_0) = l_d$$

$$g(IND2) \prec g(l_0)$$

$$in\ s : c(to), g(IND2), f(IND1); 1$$

together imply that *g(IND2)* is a trajectory that ends at the location of *f(IND1)* (third constraint) that temporally precedes the utterance location l_d (first and second constraint), see figure below. Notice that *c(to)* is a relation between a location (*g(IND2)*) and an individual (*f(IND1)*), and that the meaning of *c(to)* is given by means of the location of *f(IND1)*, which supposes that one can talk about an individual's location.



In connection with locative PP's it may be interesting to investigate which are the "primitive" types of locations (point, curve, interval in time, and so on ...) that appear in natural languages, and what the "primitive" relations between locations are.

(2) The book was lying on the table.
(with the oblique object reading)

The meaning of this sentence is a relation $[\![SIT.2]\!]$ between an utterance situation u ($= d, c$) in $MF_{[\![SIT.2]\!]}$ and a described situation s, where the relation $u[\![SIT.2]\!]s$ holds if and only if:
there exist an anchor g on *SIT.2.LOC*, i.e.

$g(l_0) = l_d$ and

$g(IND2) \prec g(l_0)$

and an extension f of g that anchors *IND1*, *IND2* and *IND5*

such that *f(IND1)* is the unique individual

such that *in s : c(book),f(IND1);1*

and *f(IND3)* is the unique individual

such that *in s : c(table),f(IND3);1*

such that

in s : c(on),f(IND5),f(IND3);1

in s : at g(IND2) : c(lie),f(IND1),f(IND5);1

Here *IND5* is anchored to a location and *c(on)* is a relation between a location and an individual which will hold when the first argument is *on* the location of the second argument. We shall not try to give an exact definition of the relation on, but rely on the intuitions of the reader. (There might also be some difficulties in spelling out such a definition: suppose a book is on a newspaper, the newspaper is on a table, an the table is *on* the floor. Is the book on the table? And, is the book on the floor?)

The relation *c(lie)* is a relation between an individual and a location. One simple definition of *c(lie)* is that the first argument's location must be the second argument, and besides, the first argument must be lying (not standing).

Remark. An alternative way of analyzing the sentence above is to consider the main relation in the sentence to be "lie-on" with arguments "the book" and "the table". But, then some of the compositionality which we have accounted for here will be lost. For every combination of lie and a preposition like *on, under, besides,* there will be a new "primitive" relation "lie-on", "lie-under", "lie-besides",..., and the number of relations continues to grow when one combines the prepositions above with verbs like *lay, live, put*, etc. However, the meaning of such a relation is understood when both the meaning of the verb and the preposition are understood. Therefore a compositional analysis is preferable. (Sometimes verbs actually combine with prepositions to form new "primitive" relations as in the sentences "Anne *looked after* Peter's goldfish" and "Peter *put on* some water".)

In addition to being compositional, the analysis given here also accounts for sentences such as "The book was lying here", where here is a one-valued relation which takes a location as argument; this argument will be the second argument of *lie*.

APPENDIX B

A LYNDON TYPE INTERPOLATION THEOREM FOR MANY-SORTED FIRST ORDER LOGIC

In this appendix we shall explore a certain relationship between one-sorted and many-sorted first order logic. As an application, we show how the Craig-Lyndon interpolation theorem for many-sorted first order logic without equality and function symbols can be derived as a corollary to its one-sorted counterpart.

To our knowledge, such an interpolation theorem has never previously been proved in the literature. Many-sorted interpolation theorems are treated extensively in Feferman (1967), but (in the many-sorted case) no differentiation is made between positive and negative occurrences of relation symbols. Presumably the methods used there could easily be extended in order to accommodate a Lyndon-type version of the theorem. However, we feel that more new insight is provided by the present strategy, than would result from a mere recapitulation of already established arguments.

1 LANGUAGES

We shall study pairs of one-sorted and many-sorted languages, between which a certain relationship has been defined. Neither language contains equality or function symbols. Both do contain the propositional constants f and t.

ρ^1 , the one-sorted language, has an infinite list of variables $x, y, z, \dots$; an infinite list of constants $a, b, c, \dots$; the 0-ary relation symbols f and t; and a countable set of relation symbols, each of which is assigned a certain arity.

$R(t_1, .., t_n)$ is an *atomic formula* of ρ^1 iff R is n-ary and $t_1, .., t_n$ are variables or constants. The *formulas* of ρ^1 are generated from the atomic formulas with the connectives $\wedge$ (binary) , $\vee$ (binary) and $\sim$ (unary) , and by applying $\forall$ and $\exists$ with respect to a single variable.

ρ^I, the many-sorted language, is defined relative to a non-empty set I of *sorts*. For each sort $i \in I$, ρ^I has an infinite list of variables of sort i. These lists are all mutually disjoint. For each i, let f_i be a bijection from the variables of ρ^1 to the variables of ρ^I of sort i. We call the variable $f_i(x)$ the i'th projection of the variable x. As an alternative notation to $f_i(x)$ we shall also write x^i.

Also, ρ^I has for each $i \in I$ an infinite list of constants of sort i. These lists are all disjoint. We extend f_i to relate constants in the same way as it does variables. Otherwise the primitive symbols of ρ^1 and ρ^I are the same. In particular, this means that ρ^I has the *same* countable set of relation symbols as ρ^1. Moreover, R is n-ary in ρ^I iff it is n-ary in ρ^1.

The rules of well formedness are different: Each n-ary relation symbol R has an associated n-tuple $< i_1, .., i_n >$ of sorts. $R(t_1, ..t_n)$ is an atomic formula of ρ^I iff for each m, $1 \leq m \leq n$, t_m is a variable or constants of sort i_m. Thus the position of a term with respect to a relation symbol uniquely identifies its sort. This fact will be central to the subsequent constructions. Otherwise ρ^1 and ρ^I have the same rules of well formedness.

2 STRUCTURES

A structure M for ρ^1 identifies a non-empty domain D, and assigns to each n-ary relation symbol R a set $R^M \subseteq D^n$, and to each constant symbol c a member c^M of D. A variable assignment B assigns to each variable a member of D.

A structure N for ρ^I identifies an indexed set $\{D_i\}_{i \in I}$ of non-empty domains, and assigns to each predicate symbol R a set $R^N \subseteq D_{i_1} X..X D_{i_n}$ where R has the associated n-tuple $< i_1, ..i_n >$ of sorts. To each constant symbol c^i it assigns a member c^{iN} of D_i. A variable assignment A assigns to each variable x^i a member of D_i.

The rules of interpretation are the standard ones for both languages.

For each structure N for ρ^I we define the corresponding *product structure* N^p for ρ^1: The domain D^p of N^p is the product $\prod_{i \in I} D_i$, i.e. the set of functions g from I into $\bigcup_{i \in I} D_i$ such that $g(i) \in D_i$.

$< g_1, .., g_n > \in R^{N^p}$ iff $< g_1(i_1), .., g_n(i_n) > \in R^N$, where $< i_1, .., i_n >$ is the n-tuple of sorts associated with R.

Finally, $c^{N^p}(i) = c^{i^N}$ for all i.

A structure for ρ^1 is a product structure iff it is *the* product structure of *some* structure for ρ^I. We write $\models_p^1 \varphi$ to express that φ is satisfied in all product structures for ρ^1 with respect to all variable assignments. Similarly, $N \models^I \varphi[A]$ means that the ρ^I formula φ is satisfied in the ρ^I structure N with respect to A. $M \models^1 \varphi[B]$ means that the ρ^1 formula φ is satisfied in the ρ^1 structure M with respect to B.

If A is a variable assignment for ρ^I in N, then for all $i \in I$ let

$$A^p(x)(i) = A(x^i)$$

A^p is a variable assignment for ρ^1 in N^p.

Moreover, for every variable assignment B for ρ^1 in N^p there is a unique variable assignment A for ρ^I in N such that B is A^p. We denote this A by B^{-p}. Note that B^{-p} is given by the equation

$$B^{-p}(x^i) \;=\; B(x)(i).$$

B^{-p} is well defined by this equation, since each variable of ρ^I is of the form x^i for exactly one pair x, i where x is a variable of ρ^1 and i is a sort.

To every one-sorted structure there corresponds a many-sorted structure where all the domains coincide with the domain of the one-sorted structure. Moreover, given any formula of a many-sorted language, it will also be a formula of a one-sorted language, where all sort distinctions are obliterated. In such a case, the formula will be satisfied in a one-sorted structure iff it is satisfied in the corresponding many-sorted structure. Hence we obtain the following (We do not write $\models^1 \varphi$ since φ is not necessarily a formula of the particular one-sorted language ρ^1 corresponding to ρ^I through the f_i):

Lemma 2.1. *For any formula φ of ρ^I , if $\models^I \varphi$ then φ is valid in one-sorted logic.*

3 RELATIONS BETWEEN MANY-SORTED STRUCTURES AND PRODUCT STRUCTURES

We shall use the following translation $\star$ from ρ^1 to ρ^I:

$$R(t_1, .., t_n)^\star \text{ is } R(f_{i_1}(t_1), .., f_{i_n}(t_n))$$

where R has the associated n-tuple $< i_1, .., i_n >$ of sorts.

$$(\varphi \wedge \psi)^\star \text{ is } (\varphi^\star \wedge \psi^\star)$$
$$(\varphi \vee \psi)^\star \text{ is } (\varphi^\star \vee \psi^\star)$$
$$(\sim \varphi)^\star \text{ is } \sim \varphi^\star$$
$$(\exists x\varphi)^\star \text{ is } \exists x^{i_1}..\exists x^{i_n}\varphi^\star$$

where $x^{i_1}, .., x^{i_n}$ are the projections of x occurring free in $\varphi^\star$. Similarly for $(\forall x\varphi)^\star$.

We can now prove a characterization of validity in product structures.

Theorem 3.1. *For all formulas φ, structures N and variable assignments A for ρ^I:*

(i) $N^p \models^1 \varphi[A^p]$ iff $N \models^I \varphi^\star[A]$

(ii) $\models^1_p \varphi$ iff $\models^I \varphi^\star$

Proof: (ii) follows from (i), plus the fact that the operation from A to A^p has an inverse. We prove (i) by induction on formulas.
For atomic formulas, the result follows immediately from the definitions of N^p and A^p. The induction steps for sentential connectives are routine.

For the induction step for $\exists$, observe that

$$N^p \models^1 \exists x\varphi[A^p]$$

iff

$$N^p \models^1 \varphi[B]$$

for a B differing with A^p at most at x. By the I.H., this holds iff

$$N \models^I \varphi^\star[A']$$

for an A' differing with A at most at the projections of x. In turn, this holds iff

$$N \models^I \exists x^{i_1}..\exists x^{i_n}\varphi^\star[A]$$

where $x^{i_1}, .., x^{i_n}$ are the projections of x occurring free in $\varphi^\star$. But this last formula is just $(\exists x\varphi)^\star$, hence the induction step is complete. The induction step for $\forall$ is similar.

An occurrence of a term t in a formula of ρ^1 is an *i-occurrence* iff the position of the occurrence is in ρ^I reserved for terms of sort i. We next observe a simple property of $\star$. It relies on the fact that all the f_i are one-to-one, and is easily verified.

Lemma 3.1. *x^i has a free occurrence in $\varphi^\star$ iff x has a free i-occurrence in φ.*

4 RELATIONS BETWEEN ONE-SORTED STRUCTURES AND PRODUCT STRUCTURES

An arbitrary formula which is valid in all product structures is not necessarily a theorem of first order logic. In this section we shall identify a subclass of formulas for which general validity does coincide with validity with respect to product structures.

We call a ρ^1 formula φ *regular* if no term has both i- and j-occurrences (free or bound) in φ for $i \neq j$:

Lemma 4.1. *Suppose φ is regular and without vacuous quantifiers. Then $\varphi^\star$ is an alphabetical variant of φ.*

Proof: For each term t (free or bound) in φ, let i_t be the unique sort i for which t has i-occurrences in φ. (There is at least one such i_t since φ has no vacuous quantifiers, and at most one such i_t since φ is regular) Now $\varphi^\star$ is obtained from φ by replacing all occurrences (free or bound) of t by $f_{i_t}(t)$. Since the f_i are all one-to-one *and* no term of ρ^I belongs to more than one sort, $\varphi^\star$ will be an alphabetical variant of φ.

Theorem 4.1. *For regular formulas φ, $\models^1_p \varphi$ iff $\models^1 \varphi$.*

Proof: Suppose $\models^1_p \varphi$. By theorem 3.1, $\models^I \varphi^\star$. By lemma 2.1 $\varphi^\star$ is also valid when considered as a formula of one-sorted first order logic. Since φ is regular, $\varphi^\star$ is just an alphabetical variant of φ, with vacuous quantifiers deleted. Hence $\models^1 \varphi$. The other direction is trivial.

5 RELATIONS BETWEEN MANY-SORTED AND ONE-SORTED STRUCTURES

Combining the two previous theorems we get the following:

Theorem 5.1. *For regular formulas φ, $\models^1 \varphi$ iff $\models^I \varphi^\star$.*

In order to translate a question about many-sorted logic into a question about one-sorted logic, theorem 5.1 by itself may be insufficient, since not all formulas of ρ^I will be $\varphi^\star$ for some φ of ρ^1, still less for a *regular* φ. Hence the following theorem is useful:

Theorem 5.2. *Given a many-sorted language ρ^I of the type in consideration, and a formula ψ of ρ^I without vacuous quantifiers. Then there exists a one-sorted language ρ^1 and functions $\{f_i\}_{i\in I}$ so that ρ^1 and ρ^I stand in the described relation through the f_i, and furthermore there exists a regular formula φ of ρ^1 such that ψ is $\varphi^\star$.*

Proof: Choose separate enumerations (without repetitions) of the variables of each sort, in such a way that no two variables of different sorts, occurring in ψ, have the same position in their respective lists. Similarly for the constants. As variables and constants of ρ^1, choose any lists $< v_n >_{n<\omega}$ and $< c_n >_{n<\omega}$ without repetitions. For the bijections f_i, let $f_i(v_n)$ be the variable with position n in the new enumeration of variables of sort i. Similarly for constants. A new formula φ is now obtained by substituting $f_i^{-1}(t)$ for each constant or variable t (free or bound) in ψ, where i is the sort of t. From the way ρ^1 and ρ^I are related, φ will be regular and $\varphi^\star = \psi$.

6 INTERPOLATION THEOREM

The following holds for many-sorted languages ρ^I of the type in consideration (A relation symbol is in $Rel^+(\alpha)$ [resp. $Rel^-(\alpha)$] iff P has a positive [resp. negative] occurrence in α, i.e. an occurrence within the scopes of evenly [resp. unevenly] many occurrences of the negation sign.):

Theorem 6.1. *If $\models^I \varphi \supset \psi$, then $\models^I \varphi \supset \chi$ and $\models^I \chi \supset \psi$ for a formula χ where $Rel^+(\chi) \subseteq Rel^+(\varphi) \cap Rel^+(\psi)$ and $Rel^-(\chi) \subseteq Rel^-(\varphi) \cap Rel^-(\psi)$.*

Proof: This is the many-sorted counterpart to the Craig-Lyndon interpolation theorem initially proved in Lyndon (1959a). The present theorem is derived more directly from the version in Henkin (1963), we prove it as a corollary to this version.

Suppose $\models^I \varphi \supset \psi$. We can also suppose that $\varphi \supset \psi$ contains no vacuous quantifiers. From theorem 5.2 we know of a one-sorted counterpart ρ^1 with a regular $\varphi' \supset \psi'$ such that $\varphi \supset \psi$ is $(\varphi' \supset \psi')^\star$.

By theorem 5.1 $\models^1 \varphi' \supset \psi'$, and by the interpolation theorem for ρ^1 there exists a formula χ with the appropriate syntactic properties, such that $\models^1 \varphi' \supset \chi$ and $\models^1 \chi \supset \psi'$. χ is not necessarily regular, but it follows that $\models^1_p \varphi' \supset \chi$ and $\models^1_p \chi \supset \psi'$. Hence by theorem 3.1 $\models^I \varphi \supset \chi^\star$ and $\models^I \chi^\star \supset \psi$ and $\chi^\star$ is our interpolant.

7 PARTIAL SEMANTICS

We next define the notion of partial models for ρ^I and ρ^1. This will be useful for the arguments in Appendix C. A partial model for ρ^I differs in only one respect from the complete models defined above: A relation symbol R with n-tuple $< i_1, ..i_n >$ of sorts is assigned a *pair* R^{M^+}, R^{M^-} of disjoint subsets of $D_{i_1}X..XD_{i_n}$ rather than a single subset. We also enrich the language with the negation symbol $\neg$. The rules of interpretation are those of L_3. The partial semantics for ρ^1 is defined similarly. We shall write ρ^I_3 and ρ^1_3 to refer to ρ^I and ρ^1 with the partial semantics. $\models^I_3 \varphi$ and $\models^1_3 \varphi$ are understood in the obvious way. $\varphi^\star$ is defined as before; we add the clause "$(\neg\varphi)^\star$ is $\neg\varphi^\star$."

Lemma 7.1. *If* $\models^1_3 \varphi$ *then* $\models^I_3 \varphi^\star$.

We give a sketch of the proof: Introducing $\models^1_{3\ p}$ to indicate validity in relation to *partial product structures*, we can obtain the equivalence between $\models^1_{3\ p} \varphi$ and $\models^I_3 \varphi^\star$ by a proof similar to the proof of theorem 3.1. As before, $\models^1_{3\ p} \varphi$ will follow trivially from $\models^1_3 \varphi$. These two results will take care of lemma 7.1.

APPENDIX C

PROOF OF THE RELATIVE SATURATION LEMMA

In this appendix we prove the relative saturation lemma (φ' is saturated relative to ψ in L_3 if $L_3 \vdash \psi \supset\!\sim \neg\varphi'$ implies $L_3 \vdash \psi \supset \varphi'$):

For every two pure formulas ψ and φ, there exists a pure formula φ' which is saturated relative to ψ in L_3, such that $L_3 \vdash \neg\varphi \equiv \neg\varphi'$.

The system L_3 is many-sorted (*two*-sorted; I = {individuals, locations}) and contains the special axioms for $\prec$. The ideas involved in the relative saturation lemma have little to do with these features, and we believe that it will improve readability if we first obtain the relative saturation lemma for a system with a simpler structure. Then afterwards it will take very little effort to deduce the corresponding result for L_3. Thus for the next few pages we shall concentrate on the system ρ_3^1 defined at the end of Appendix B. The relative saturation lemma is a result about *pure* formulas, and we shall use *formula* and *pure formula* synonymously. The greek letters φ, ψ, etc. will range over pure formulas. However, we shall still employ such *extended* formulas as $\psi \supset\!\sim \varphi$ to express relations between pure formulas ψ, φ.

1 THE RELATIVE SATURATION LEMMA FOR QUANTIFIER FREE FORMULAS

In the case of quantifier free formulas of ρ_3^1, we can state a stronger version of the relative saturation lemma:

Lemma 1.1. *For every quantifier free formula φ there is a quantifier free formula φ' such that $\models_3^1 \neg\varphi \equiv \neg\varphi'$ and for every (not necessarily quantifier free) formula ψ, if $\models^1 \psi \supset \varphi'$ then $\models_3^1 \psi \supset \varphi'$.*

To prove this, we shall define a syntactic function PS on quantifier free formulas in conjunctive normal form. We need the following lemma:

Lemma 1.2. *Every quantifier free formula is strongly equivalent to a quantifier free formula in conjunctive normal form.*

Proof: Using the equivalences

$$\models_3^1 \neg\neg\varphi \rightleftharpoons \varphi$$

$$\models_3^1 \neg(\varphi \wedge \psi) \rightleftharpoons (\neg\varphi \vee \neg\psi)$$

$$\models_3^1 \neg(\varphi \vee \psi) \rightleftharpoons (\neg\varphi \wedge \neg\psi)$$

we can find a formula in $\neg$ normal form which is strongly equivalent to a given formula. The rest follows by repeated applications of

$$\models_3^1 (\varphi \vee (\psi \wedge \chi)) \rightleftharpoons ((\varphi \vee \psi) \wedge (\varphi \vee \chi)).$$

Definition. By a *literal* we mean an atomic or negated atomic formula.

Definition. Let φ be a quantifier free formula in conjunctive normal form. We define the formula $PS(\varphi)$ recursively as follows:
(i) If φ is a disjunction (n-ary) of literals, then
- $PS(\varphi)$ is t if φ contains both an atomic formula and its negation.
- $PS(\varphi)$ is φ otherwise.

(ii) $PS(\varphi \wedge \psi)$ is $(PS(\varphi) \wedge PS(\psi))$.

Lemma 1.1 now follows from lemmas 1.2 , 1.3 and 1.4:

Lemma 1.3. *Let φ be a quantifier free formula in conjunctive normal form. Then $\models_3^1 \neg\varphi \equiv \neg PS(\varphi)$.*

Proof: This follows by a simple induction.

Lemma 1.4. *Let φ be a quantifier free formula in conjunctive normal form. If $\models^1 \psi \supset PS(\varphi)$, then $\models_3^1 \psi \supset PS(\varphi)$.*

Proof: We prove this by induction on quantifier free formulas in conjunctive normal form
Basis: φ is a disjunction of literals.

If φ contains both a formula and its negation, then φ is negatively equivalent to t, which is $PS(\varphi)$. Trivially, t is saturated relative to every other formula.

If φ does *not* contain both an atomic formula and its negation, then φ is itself saturated relative to every other formula. To see this, suppose φ is $\alpha_1 \vee .. \vee \alpha_n$ where $\alpha_1, .., \alpha_n$ are literals, and suppose for the formula

ψ that *not* $\models^1_3 \psi \supset (\alpha_1 \vee .. \vee \alpha_n)$. Then in some model, with respect to some variable assignment, ψ is true and none of $\alpha_1, .., \alpha_n$ is true. Since the language does not contain an identity symbol, we may assume that with the given variable assignment all constants and variables denote different elements. Then since the α_i are mutually non-contradicting literals, we can extend this model to one where all the α_i are false. In this model, ψ is still true by persistence, so we have a counterexample to $\models^1_3 \psi \supset (\alpha_1 \vee .. \vee \alpha_n)$. By contraposition, $\models^1 \psi \supset (\alpha_1 \vee .. \vee \alpha_n)$ implies $\models^1_3 \psi \supset (\alpha_1 \vee .. \vee \alpha_n)$.

Induction step: If φ is $\varphi_0 \wedge \varphi_1$, then $PS(\varphi)$ is $PS(\varphi_0) \wedge PS(\varphi_1)$. But a simple argument shows that any conjunction of saturated formulas is itself saturated.

2 A SEQUENT CALCULUS

To prove the relative saturation lemma for ρ^1_3, we shall take advantage of the proof-theoretical properties of a Gentzen sequent calculus style system.

"$\Gamma \models^1 \Delta$" says that in every ρ^1 model and variable assignment where all members of Γ are satisfied, at least one member of Δ is satisfied. The following system is intended to characterize $\Gamma \models^1 \Delta$ for finite sequences Γ and Δ of prenex formulas:

Axioms of $\vdash$

$\Gamma \vdash \Delta$ is an axiom iff all formulas of Γ and Δ are quantifier free and $\Gamma \models^1 \Delta$.

Inference rules of $\vdash$

Thinning.

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, \varphi}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, \varphi \vdash \Delta}$$

Interchange.

$$\frac{\Gamma \vdash \Delta_0, \varphi, \psi, \Delta_1}{\Gamma \vdash \Delta_0, \psi, \varphi, \Delta_1}$$

$$\frac{\Gamma_0, \varphi, \psi, \Gamma_1 \vdash \Delta}{\Gamma_0, \psi, \varphi, \Gamma_1 \vdash \Delta}$$

Contraction.

$$\frac{\Gamma \vdash \Delta, \varphi, \varphi}{\Gamma \vdash \Delta, \varphi}$$

$$\frac{\Gamma, \varphi, \varphi \vdash \Delta}{\Gamma, \varphi \vdash \Delta}$$

In $R\exists$, $L\forall$, $R\forall$ and $L\exists$, s is any variable or constant symbol. x must not occur free in φ inside the scope of a quantifier binding s. In $R\forall$ and $L\exists$, s must not occur free in the resultant sequent. (Every occurrence of a constant symbol is considered free.)

$R\forall$

$$\frac{\Gamma \vdash \Delta, \varphi(s/x)}{\Gamma \vdash \Delta, \forall x\varphi}$$

$R\exists$

$$\frac{\Gamma \vdash \Delta, \varphi(s/x)}{\Gamma \vdash \Delta, \exists x\varphi}$$

$L\forall$

$$\frac{\Gamma, \varphi(s/x) \vdash \Delta}{\Gamma, \forall x\varphi \vdash \Delta}$$

$L\exists$

$$\frac{\Gamma, \varphi(s/x) \vdash \Delta}{\Gamma, \exists x\varphi \vdash \Delta}$$

Lemma 2.1. *If* $\Gamma \vdash \Delta$ *then* $\Gamma \models^1 \Delta$.

Proof: Routine.

In important respects $\vdash$ resembles Gentzen's system LK. The reader who consults the original text of Gentzen (e.g. in Szabo (1969)), will immediately recognize that completeness of $\vdash$ is just a reformulation of Gentzen's *Sharpened Hauptsatz* or *midsequent theorem*, modulo one difference in the languages studied: The language for LK does not include the special logical symbols f and t. Hence from the midsequent theorem for LK we can only deduce completeness of $\vdash$ for formulas not containing t and f.

However, note that from $\Gamma \models^1 \Delta$ we can deduce $\Gamma', p^t \models^1 \Delta', p^f$, where p^t and p^f are distinct 0-ary relation symbols not occurring in Γ or Δ, and Γ' and Δ' are obtained from Γ and Δ by substitution of p^t

and p^f for t and f respectively. Hence from $\Gamma \models^1 \Delta$ we can deduce $\Gamma', p^t \vdash \Delta', p^f$. Substituting back throughout the derivation, we also get $\Gamma, t \vdash \Delta, f$. Now, a simple induction will show that $\Gamma, t \vdash \Delta, f$ always implies $\Gamma \vdash \Delta$. Hence we can state the following:

Lemma 2.2. *If Γ and Δ consist of prenex formulas, then $\Gamma \models^1 \Delta$ and $\Gamma \vdash \Delta$ are equivalent.*

In $\vdash$ the rule of thinning is in fact superfluous: A straightforward proof shows that if $\Gamma \vdash \Delta$ follows by a thinning-free derivation, then so do $\Gamma \vdash \Delta, \varphi$ and $\Gamma, \varphi \vdash \Delta$.

For expository purposes it will be convenient to leave out thinning. We write $|\!\vdash$ for $\vdash$ without thinning, and $\Gamma \;|\!\vdash \Delta$ for sequents in this system:

Lemma 2.3. *If Γ and Δ consist of prenex formulas, then $\Gamma \models^1 \Delta$ and $\Gamma \;|\!\vdash \Delta$ are equivalent.*

We shall use these axiomatizations to prove the relative saturation lemma in the following way: If $\models^1 \psi \supset \varphi$ then $\psi \;|\!\vdash \varphi$. Roughly, we shall then show how to construct an appropriate saturant of φ relative to ψ, using a derivation of $\psi \;|\!\vdash \varphi$. However, this method breaks down at steps of contraction on the right side. Consequently we shall also define a weaker system *without* contraction on the right side, and prove a relation between the two (three) systems: Let $||\!\vdash$ be $|\!\vdash$ without contraction on the right. We write $\Gamma \;||\!\vdash \Delta$ for sequents in this system. $||\!\vdash$ is strictly weaker than $|\!\vdash$, but there is still a certain sense in which the two are equivalent. This requires some definitions, we start with the notion of *prenex disjunctions*:

Definition. χ is a *prenex disjunction* of φ and ψ if and only if this follows from (1) and (2):

(1) If φ and ψ are quantifier free, then $\varphi \vee \psi$ is a prenex disjunction of φ and ψ.

(2) If χ is a prenex disjunction of φ and ψ, and x does not occur in φ, then $\exists x\chi$ is a prenex disjunction of φ and $\exists x\psi$, and $\forall x\chi$ is a prenex disjunction of φ and $\forall x\psi$. Similarly, if x does not occur in ψ, then $\exists x\chi$ is a prenex disjunction of $\exists x\varphi$ and ψ, and $\forall x\chi$ is a prenex disjunction of $\forall x\varphi$ and ψ.

Lemma 2.4.

(i) If χ is a prenex disjunction of φ and ψ, then χ is strongly equivalent to $\varphi \vee \psi$.

(ii) If χ is a prenex disjunction of φ and ψ, then the free variables and constants of χ are exactly the free variables and constants of $\varphi \vee \psi$.

(iii) Suppose x does not occur in φ, and there is no quantifier binding s in ψ. If χ is a prenex disjunction of φ and $\psi(s/x)$, then χ is $\chi'(s/x)$ for a prenex disjunction χ' of φ and ψ.

Proof: The proofs of (i) and (ii) are standard. For (iii), let the matrix of χ' be the disjunction of the matrices of φ and ψ, and let the quantifier prefix of χ' be the quantifier prefix of χ. A simple induction will show that this χ' has the properties described.

Lemma 2.5. *Suppose φ and ψ are prenex, and no bound variable in φ has any occurrence in ψ, and vice versa. Further suppose $\Gamma \mid\mid\!\vdash \Pi$, where Π is a permutation of Δ, φ, ψ. Then there exists a prenex disjunction χ of φ and ψ such that $\Gamma \mid\mid\!\vdash \Delta, \chi$.*

Proof: By induction on derivations in $\mid\mid\!\vdash$.

Basis: If all formulas of Γ and Π are quantifier free and $\Gamma \mid\mid\!\vdash \Pi$, then $\Gamma \mid\mid\!\vdash \Delta, (\varphi \vee \psi)$ is an axiom.

Induction step: The steps for all the left rules are straightforward. The step for interchange on the right is trivial by the formulation of the lemma. The steps for $R\forall$ and $R\exists$ are similar to each other; we consider $R\forall$.

(case 1) If neither φ nor ψ was the active formula of the last step, then we may assume that the last step was of the form

$$\frac{\Gamma \mid\mid\!\vdash \varphi, \psi, \Delta_0, \alpha(s/x)}{\Gamma \mid\mid\!\vdash \varphi, \psi, \Delta_0, \forall x \alpha}$$

where Δ is $\Delta_0, \forall x \alpha$. By I.H. and interchange there is a derivation of

$$\Gamma \mid\mid\!\vdash \chi, \Delta_0, \alpha(s/x)$$

where χ is a prenex disjunction of φ and ψ. Now every variable or constant occurring free in χ will also occur free in φ or ψ; hence $R\forall$ applies again, and we get

$$\Gamma \mid\mid\!\vdash \chi, \Delta_0, \forall x \alpha$$

and by interchange

$$\Gamma \mid\mid\!\vdash \Delta, \chi.$$

(case 2) If ψ was the active formula in the last step (the proof is similar for φ), then we may assume that the last step was of the form

$$\frac{\Gamma \mid\mid\!\vdash \Delta, \varphi, \psi_0(s/x)}{\Gamma \mid\mid\!\vdash \Delta, \varphi, \forall x \psi_0}$$

Here s has no free occurrences in the resulting sequent. We may also assume that s has no bound occurrences in φ. Now by the induction hypothesis there is a derivation of

$$\Gamma \mid\mid\!\vdash \Delta, \chi$$

where χ is a prenex disjunction of φ and $\psi_0(s/x)$. By the assumption of the lemma, x does not occur in φ. Hence (by lemma 2.4(iii)) χ is $\chi'(s/x)$, where χ' is a prenex disjunction of φ and ψ. By applicability of $R\forall$, s does not occur free in φ or $\forall x \psi$, so (by lemma 2.4(ii)) neither will s occur free in $\forall x \chi'$. Hence $R\forall$ applies again; from

$$\Gamma \mid\mid\!\vdash \Delta, \chi'(s/x)$$

we get

$$\Gamma \mid\mid\!\vdash \Delta, \forall x \chi'.$$

Clearly $\forall x \chi'$ is a prenex disjunction of φ and $\forall x \psi_0$.

We proceed to define *prenex multiples*, but first we note a simple fact about bound alphabetical variants:

Lemma 2.6. *If* $\Gamma \mid\mid\!\vdash \Delta, \varphi$ *then also* $\Gamma \mid\mid\!\vdash \Delta, \varphi'$ *if* φ' *is a bound alphabetical variant of* φ.

Proof: Routine.

Definition. We define the notion of *prenex multiples* as follows:

(1) If φ is a prenex formula, then φ itself is a prenex multiple of φ.

(2) If φ' is a prenex multiple of φ, and φ'' is a bound alphabetical variant of φ', then φ'' is a prenex multiple of φ.

(3) If φ' and φ'' are prenex multiples of φ such that no bound variable of φ' occurs in φ'' and vice versa, and χ is a prenex disjunction of φ' and φ'', then χ is a prenex multiple of φ.

(4) If φ' is a prenex multiple of φ then $\exists x\varphi'$ is a prenex multiple of $\exists x\varphi$, and $\forall x\varphi'$ is a prenex multiple of $\forall x\varphi$.

The next lemma is easily verified, although part (iii) is somewhat tedious:

Lemma 2.7.

(i) If φ' is a prenex multiple of φ, then φ' and φ are strongly equivalent.

(ii) If φ' is a prenex multiple of φ, then the free variables and constants of φ' are exactly the free variables and constants of φ.

(iii) Suppose that s is not bound in φ, and suppose also that x is not a bound variable of χ. If χ is a prenex multiple of $\varphi(s/x)$, then χ is $\chi'(s/x)$ for some prenex multiple χ' of φ.

We can now state the relation between $|\vdash$ and $||\vdash$:

Lemma 2.8. *If $\Gamma \mid\vdash \varphi_1, .., \varphi_n$ then for some formulas $\varphi'_1, .., \varphi'_n$ we have $\Gamma \mid\mid\vdash \varphi'_1, .., \varphi'_n$ where each φ'_i is a prenex multiple of φ_i.*

Proof: By induction on derivations in $|\vdash$.

Basis: If $\Gamma \mid\vdash \varphi_1, .., \varphi_n$ is an axiom, then so is $\Gamma \mid\mid\vdash \varphi_1, .., \varphi_n$.

Induction step:

The left rules are straightforward; we consider the right rules. The step for interchange on the right is trivial.

For contraction, suppose the last step is

$$\frac{\Gamma \mid\vdash \varphi_1, .., \varphi_{n-1}, \varphi_n, \varphi_n}{\Gamma \mid\vdash \varphi_1, .., \varphi_{n-1}, \varphi_n}$$

Then by the induction hypothesis we can derive

$$\Gamma \mid\mid\vdash \varphi'_1, .., \varphi'_{n-1}, \varphi'_n, \varphi''_n$$

where φ'_i is a prenex multiple of φ_i, and φ''_n is a prenex multiple of φ_n. By lemma 2.6 and (2) in the definition of "prenex multiple" we can suppose that no bound variable in φ'_n occurs in φ''_n and vice versa. By lemma 2.5 there is a derivation of

$$\Gamma \mid\mid\vdash \varphi'_1, .., \varphi'_{n-1}, \varphi'''_n$$

where φ_n''' is a prenex disjunction of φ_n' and φ_n'', and hence a prenex multiple of φ_n.

For $R\exists$ (the step for $R\forall$ is similar), suppose the last step is

$$\frac{\Gamma \Vdash \varphi_1, .., \varphi_{n-1}, \varphi_n(s/x)}{\Gamma \Vdash \varphi_1, .., \varphi_{n-1}, \exists x \varphi_n}$$

By the induction hypothesis there is a derivation of

$$\Gamma \mathrel{||\!\vdash} \varphi_1', .., \varphi_{n-1}', \chi$$

where φ_i' is a prenex multiple of φ_i, and χ is a prenex multiple of $\varphi_n(s/x)$. By lemma 2.6 and (2) in the definition of "prenex multiple", we may assume that x is not a bound variable of χ. Then by lemma 2.7(iii) χ is $\chi'(s/x)$ for a prenex multiple χ' of φ_n. From

$$\Gamma \mathrel{||\!\vdash} \varphi_1', .., \varphi_{n-1}', \chi'(s/x),$$

$$\Gamma \mathrel{||\!\vdash} \varphi_1', .., \varphi_{n-1}', \exists x \chi'$$

follows by $R\exists$. Clearly $\exists x \chi'$ is a prenex multiple of $\exists x \varphi_n$. This completes the proof.

For an example of what lemma 2.8 gives us, consider the formula φ:

$$\exists x \forall y (R(x) \vee \neg R(y)).$$

We have $\Vdash \varphi$, but not $\mathrel{||\!\vdash} \varphi$. Still, there is a prenex multiple φ' of φ such that $\mathrel{||\!\vdash} \varphi'$. The formula

$$\exists x \forall y \exists x' \forall y' (R(x) \vee \neg R(y) \vee R(x') \vee \neg R(y'))$$

is an example.

3 PROOF OF THE RELATIVE SATURATION LEMMA.

We can now finally prove the relative saturation lemma. In order for the induction to work, we prove something a little stronger. Let $ft(\varphi)$ be the set of terms with free occurrences in φ:

Lemma 3.1. *If* $\Gamma \mid\mid\!\vdash \varphi$ *then there exists a formula* χ *satisfying*
(i) $\Gamma \models_3^1 \chi$
(ii) $\models_3^1 \neg\varphi \equiv \neg\chi$
(iii) $ft(\chi) \subseteq ft(\varphi)$

Proof: By induction on the derivation of sequents $\Gamma \mid\mid\!\vdash \varphi$.

Basis: If $\Gamma \mid\mid\!\vdash \varphi$ is an axiom, then Γ and φ are quantifier free. Now φ is strongly equivalent to a quantifier free formula φ' in conjunctive normal form, such that $ft(\varphi') \subseteq ft(\varphi)$. Since $\Gamma \models^1 \varphi'$, from lemmas 1.3 and 1.4 it is seen that $PS(\varphi')$ satisfies properties (i) and (ii) of the χ. Moreover, since $PS(\varphi')$ is obtained from φ' by substituting t for certain subformulas, $PS(\varphi')$ satisfies also the third property.

Induction step: Aside from the four quantificational rules, there are three structural rules in $\mid\mid\!\vdash$; contraction on the left, and interchange on both sides. Interchange and contraction on the left are straightforward, while interchange on the right does not apply in this case since the consequent is a singleton. We consider the quantificational rules.

$L\exists$ The last step is of the form

$$\frac{\Gamma_0, \psi(s/x) \mid\mid\!\vdash \varphi}{\Gamma_0, \exists x\psi \mid\mid\!\vdash \varphi}$$

By the I.H.

$$\Gamma_0, \psi(s/x) \models_3^1 \chi$$

for a χ such that $\models_3^1 \neg\varphi \equiv \neg\chi$ and $ft(\chi) \subseteq ft(\varphi)$. By the applicability of $L\exists$, s does not occur free in Γ_0, $\exists x\psi$ or φ. Hence neither does s occur free in χ. Consequently

$$\Gamma_0, \exists x\psi \models_3^1 \chi.$$

$R\forall$ The last step is of the form

$$\frac{\Gamma \mid\mid\!\vdash \varphi(s/x)}{\Gamma \mid\mid\!\vdash \forall x\varphi}$$

By the I.H.

$$\Gamma \models_3^1 \chi$$

for a χ such that $\models_3^1 \neg\varphi(s/x) \equiv \neg\chi$ and $ft(\chi) \subseteq ft(\varphi(s/x))$.

Since bound alphabetical variants are strongly equivalent, we can suppose that x is substitutable for s in χ. Also, since x has no free occurrences in χ, χ is identical to $\chi(x/s)(s/x)$. Hence

$$\Gamma \models_3^1 \forall x(\chi(x/s)).$$

Since s has free occurrences in neither of $\forall x\varphi$ or $\forall x(\chi(x/s))$, from

$$\models_3^1 \neg\varphi(s/x) \equiv \neg\chi(x/s)(s/x)$$

we can deduce

$$\models_3^1 \neg\forall x\varphi \equiv \neg\forall x(\chi(x/s)).$$

Since $ft(\chi) \subseteq ft(\varphi(s/x))$, also $ft(\forall x(\chi(x/s))) \subseteq ft(\forall x\varphi)$.

$L\forall$ This is similar to the induction step for $L\exists$; it is even a little simpler.

$R\exists$ The last step is of the form

$$\frac{\Gamma \mathrel{||\!\!-} \varphi(s/x)}{\Gamma \mathrel{||\!\!-} \exists x\varphi}$$

By the I.H.

$$\Gamma \models_3^1 \chi$$

for a χ such that $\models_3^1 \neg\varphi(s/x) \equiv \neg\chi$ and $ft(\chi) \subseteq ft(\varphi(s/x))$. Again we can assume that χ is $\chi(x/s)(s/x)$. Hence it follows directly that

$$\Gamma \models_3^1 \exists x(\chi(x/s)),$$

and so

$$\Gamma \models_3^1 \exists x(\varphi \vee \chi(x/s)).$$

Since

$$ft(\chi) \subseteq ft(\varphi(s/x)),$$

also

$$ft(\exists x(\varphi \vee \chi(x/s))) \subseteq ft(\exists x\varphi).$$

We must show that

$$\models_3^1 \neg\exists x\varphi \equiv \neg\exists x(\varphi \vee \chi(x/s)),$$

i.e.

$$\models_3^1 \forall x \neg\varphi \equiv \forall x(\neg\varphi \wedge \neg\chi(x/s)).$$

One direction is trivial here; we need to show that

$$\models_3^1 \forall x \neg\varphi \supset \forall x(\neg\chi(x/s)).$$

We *do* have

$$\models_3^1 \neg\varphi(s/x) \supset \neg\chi(x/s)(s/x);$$

hence

$$\models_3^1 \forall x \neg\varphi \supset \neg\chi(x/s)(s/x).$$

But since s is not free in $\neg\chi(x/s)$ and not in $\forall x \neg\varphi$, this implies

$$\models_3^1 \forall x \neg\varphi \supset \forall x(\neg\chi(x/s)).$$

We can now state the relative saturation lemma for ρ_3^1:

Lemma 3.2. *For every two pure formulas φ and ψ of ρ_3^1 there exists a pure formula χ such that $\models_3^1 \neg\varphi \equiv \neg\chi$, and $\models^1 \psi \supset \chi$ implies $\models_3^1 \psi \supset \chi$.*

Proof: If it is not the case that $\models^1 \psi \supset \varphi$, use φ itself as saturant. Otherwise $\psi \models^1 \varphi$ and so $\psi' \models^1 \varphi'$ and hence $\psi' \Vdash \varphi'$ for prenex formulas ψ' and φ', strongly equivalent to ψ and φ respectively. By lemmas 2.8 and 2.7(i), also $\psi' \mid\Vdash \varphi''$ for a prenex formula φ'' strongly equivalent to φ. By lemma 3.1 $\psi \models_3^1 \chi$ for a χ such that $\models_3^1 \neg\varphi \equiv \neg\chi$.

4 THE RELATIVE SATURATION LEMMA FOR ρ_3^I

Lemma 4.1. *For every two pure formulas φ and ψ of ρ_3^I there exists a pure formula χ such that $\models_3^I \neg\varphi \equiv \neg\chi$, and $\models^I \psi \supset \chi$ implies $\models_3^I \psi \supset \chi$.*

Proof: If it is not the case that $\models^I \psi \supset \varphi$, use φ itself as saturant. Suppose $\models^I \psi \supset \varphi$. We can suppose that ψ and φ do not contain vacuous quantifiers. Now, $\psi \supset \varphi$ is defined as $\sim \psi \vee \varphi$ and is therefore not a pure formula. $\neg\psi \vee \varphi$ is pure, however, and from $\models^I \psi \supset \varphi$ we can deduce $\models^I \neg\psi \vee \varphi$. By theorem 5.2 of Appendix B there is a one-sorted language ρ^1 and functions f_i such that ρ^1 and ρ^I stand in the appropriate relation through the f_i, and such that $\neg\psi \vee \varphi$ is $\neg\psi'^{\star} \vee \varphi'^{\star}$ for regular $\neg\psi' \vee \varphi'$. By theorem 5.1 of Appendix B, $\models^1 \neg\psi' \vee \varphi'$. Hence also $\models^1 \psi' \supset \varphi'$. By lemma 3.2 there exists a χ such that $\models_3^1 \psi' \supset \chi$ and $\models_3^1 \neg\varphi' \equiv \neg\chi$. By lemma 7.1 of Appendix B, $\models_3^I \psi \supset \chi^\star$ and $\models_3^I \neg\varphi \equiv \neg\chi^\star$. So $\chi^\star$ is our relative saturant.

5 THE RELATIVE SATURATION LEMMA FOR L_3

Lemma 5.1. *For every two pure formulas φ and ψ of L_3 there exists a pure formula χ such that $L_3 \vdash \neg\varphi \equiv \neg\chi$, and $L_3 \vdash \psi \supset\!\sim \neg\chi$ implies $L_3 \vdash \psi \supset \chi$.*

Proof: The language of L_3 is a two-sorted special case of ρ^I. L_3 models can be viewed as ρ^I_3 models which satisfy the (universal closures of) the three axioms

A22. $\sim(\beta_1 \prec \beta_2) \equiv \neg(\beta_1 \prec \beta_2)$

A23. $\sim(\beta \prec \beta)$

A24. $((\beta_1 \prec \beta_2) \wedge (\beta_3 \prec \beta_4)) \supset ((\beta_1 \prec \beta_4) \vee (\beta_3 \prec \beta_2))$

None of these formulas are pure, but the disjunction of their universal closures is equivalent (in ρ^I_3) to the conjunction of the universal closures of

A22'. $(\beta_1 \prec \beta_2) \vee \neg(\beta_1 \prec \beta_2)$

A23'. $\neg(\beta \prec \beta)$

A24'. $(\neg(\beta_1 \prec \beta_2) \vee \neg(\beta_3 \prec \beta_4)) \vee ((\beta_1 \prec \beta_4) \vee (\beta_3 \prec \beta_2))$

Let λ be the conjunction of the universal closures of these. λ is pure, and by the completeness result for L_3 , $L_3 \vdash \varphi$ iff $\models^I_3 \lambda \supset \varphi$.

Now suppose $L_3 \vdash \psi \supset\!\sim \neg\varphi$, where both ψ and φ are pure formulas. Then $\models^I_3 (\lambda \wedge \psi) \supset\!\sim \neg\varphi$, hence $\models^I (\lambda \wedge \psi) \supset \varphi$. By lemma 4.1 there is a pure formula χ such that $\models^I_3 (\lambda \wedge \psi) \supset \chi$ and $\models^I_3 \neg\varphi \equiv \neg\chi$. Hence $L_3 \vdash \psi \supset \chi$ and $L_3 \vdash \neg\varphi \equiv \neg\chi$, so χ is our relative saturant.

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